Adjoint-based deterministic Inversion for Ice Sheets

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joint work with:

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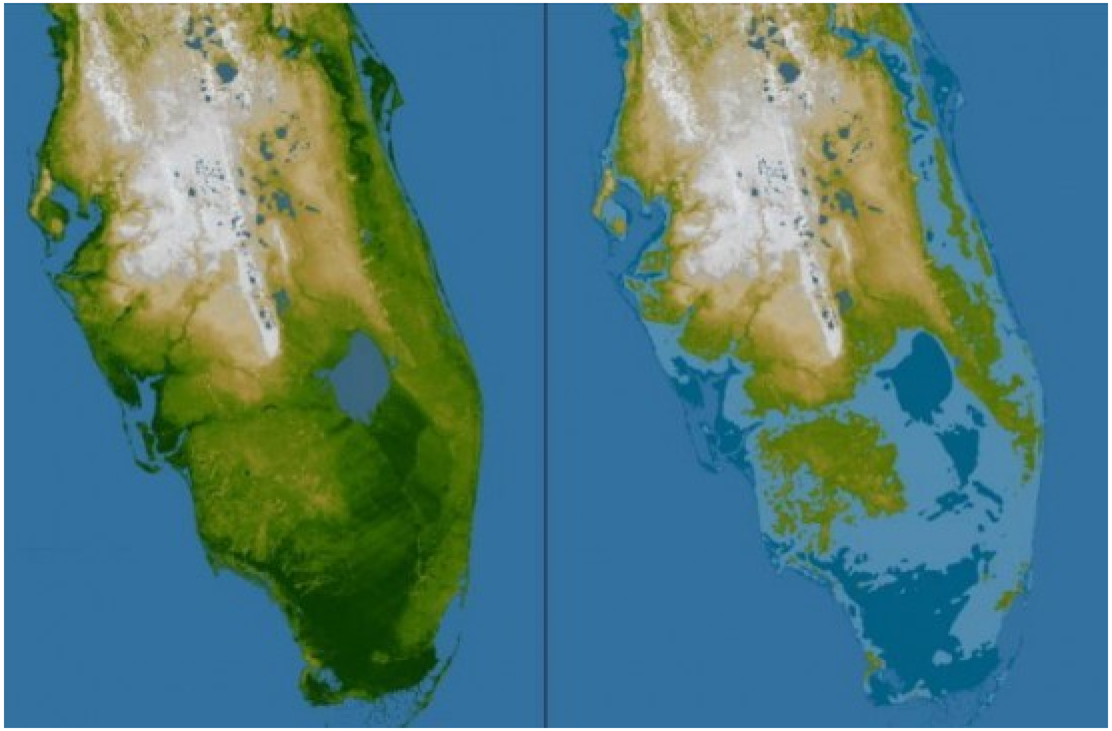
Motivations

- Glaciers and ice sheets influence the global climate, and vice-versa
- Melting of land ice determines the **sea level rise**
  melting of the Greenland ice sheet: 7 m
  melting of the Antarctic ice sheet: 61 m



South Florida projection for a sea levels rise
of 5m (dark blue) and 10m (light blue)

Ice Sheet Modeling

Main components of an ice model:

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} \rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) - \nabla\cdot\sigma = \rho\mathbf{g} \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

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with:

$$\sigma = 2\mu \mathbf{D} - \Phi I, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Non linear viscosity:

$$\mu = \frac{1}{2}\alpha(T)\,|\mathbf{D}(\mathbf{u})|^{(p-2)}, \quad p \in (1,2] \quad (\text{tipically } p \simeq \frac{4}{3})$$

Viscosity is singular when ice is not deforming

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- **Model for the evolution of the boundaries**
(thickness evolution equation)

$$\frac{\partial H}{\partial t} = H_{flux} - \nabla \cdot \int_z \mathbf{u}\, dz$$

- **Temperature equation**

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) - \rho c \mathbf{u} \cdot \nabla T + 2\dot{\varepsilon}\sigma$$

- **Coupling with other climate components (e.g. ocean, atmosphere)**

Stokes Approximations

"Reference" model: **STOKES**[1]

$O(\delta^2)$ ***FO,*** Blatter-Pattyn first order model[2] (3D PDE, in horizontal velocities)

$O(\delta)$ Zeroth order, depth integrated models:
SIA, Shallow Ice Approximation (slow sliding regimes) ,
SSA Shallow Shelf Approximation (2D PDE) (fast sliding regimes)

$\simeq O(\delta^2)$ Higher order, depth integrated (2D) models: ***L1L2***[3], (L1L1)...

$\delta :=$ ratio between ice thickness and ice horizontal extension

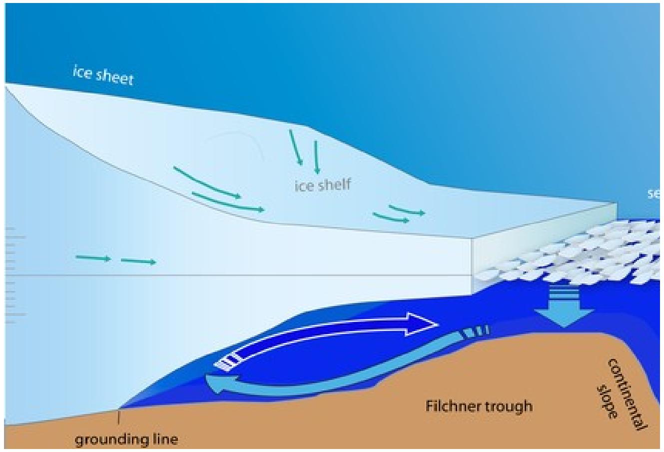
[1]Gagliardini and Zwinger, 2008. The Cryosphere.

[2]Dukowicz, Price and Lipscomb, 2010. J. Glaciol.

[3]Schoof and Hindmarsh, 2010. Q. J. Mech. Appl. Math.

(Numerical) Modeling Issues

- Computationally challenging, due to complexity of models, of geometries and large domains
  - design of linear/nonlinear solvers, preconditioners, etc.
  - mesh adaptivity especially close to the grounding line.



- Boundary conditions / coupling (e.g. with ocean)
  - Floating/calving
  - Basal friction at the bedrock,
  - Subglacial hydrology,
  - Heat exchange / phase change.

- Initialization / parameter estimation.

- Uncertainty quantification.

Inverse Problem

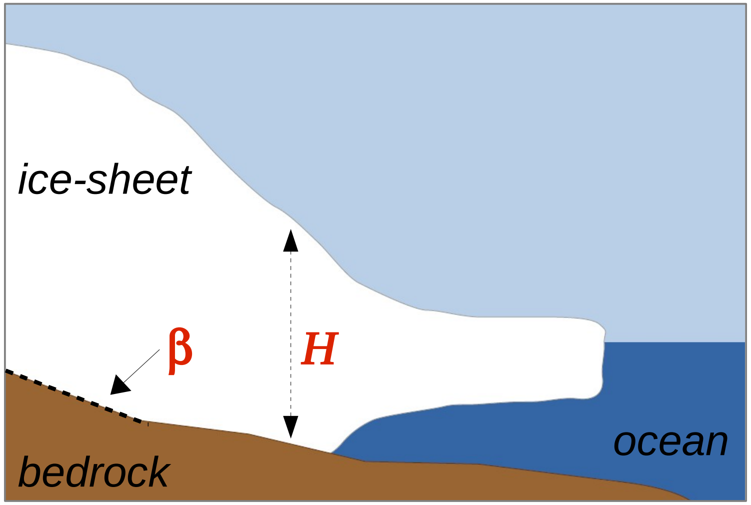
Estimation of ice-sheet initial state*

(w/ G. Stadler, UT, and S. Price, LANL)

Problem: **what is the initial thermo-mechanical state of the ice sheet?**

Available data/measurements:

- *ice extension and surface topography*
- ***surface velocity***
- ***Surface Mass Balance*** *(SMB: accumulation/melt rate)*
- ***ice thickness H (very noisy)***



Fields to be estimated :

- ***ice thickness H***
- ***basal friction β***

Additional information:

- *ice fulfills* ***nonlinear Stokes equation***
- *ice is almost* ***at thermo-mechanical equilibrium***

Assumption (for now):

- *given* ***temperature field***

Perego, Price, Stadler, JGR 2014

Inverse Problem

Estimation of ice-sheet initial state

Steady State equations and basal sliding conditions

How to prescribe ice-sheet mechanical equilibrium:

divergence flux

$$\frac{\partial H}{\partial t} = -\mathrm{div}\,(\mathbf{U}H) + \tau_s, \qquad \mathbf{U} = \frac{1}{H}\int_z \mathbf{u}\,dz.$$

Surface Mass Balance

At equilibrium: $\boxed{\mathrm{div}\,(\mathbf{U}H) = \tau_s}$

Boundary condition at ice-bedrock interface:

$$(\sigma\mathbf{n} + \beta\mathbf{u})_{||} = \mathbf{0} \quad on \quad \Gamma_\beta$$

Bibliography*:

Arthern, Gudmundsson, J. Glaciology. 2010
Price, Payne, Howat and Smith, PNAS 2011
Brinkerhoff et al., Annals of Glaciology, 2011
Morlighem et al. Geophysical Research Letters, 2013
Pollard DeConto, TCD 2012
Petra, Zhu, Stadler, Hughes, Ghattas, J. Glaciology , 2012.
Goldberg and Heimbach, The Cryosphere 2013.
Michel et al., Computers & Geosciences, 2014.

Inverse Problem

Estimation of ice-sheet initial state

PDE-constraint optimization problem: cost functional

Problem: find initial conditions such that the ice is almost at thermo-mechanical equilibrium given the geometry and the SMB, and matches available observations.

Optimization Problem:
find β and H that minimizes the functional $\mathcal{J}$

$$
\begin{aligned}
\mathcal{J}(\beta, H) &= \int_\Sigma \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds && \text{surface velocity mismatch} \\
&+ \int_\Sigma \frac{1}{\sigma_\tau^2} |\mathrm{div}(\boldsymbol{U} H) - \tau_s|^2 \, ds && \text{SMB mismatch} \\
&+ \int_\Sigma \frac{1}{\sigma_H^2} |H - H^{obs}|^2 \, ds && \text{thickness mismatch} \\
&+ \mathcal{R}(\beta, H) && \text{regularization terms.}
\end{aligned}
$$

Common

Proposed

subject to ice sheet model equations
(FO or Stokes)

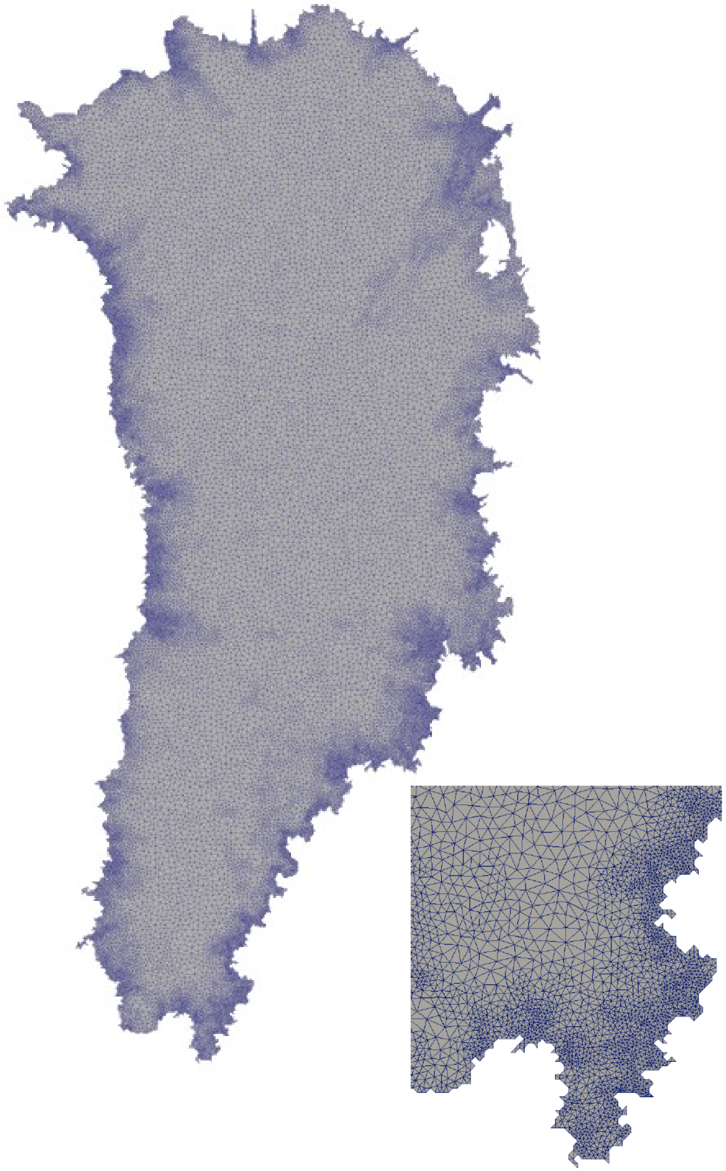
$\boldsymbol{U}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_s: SMB
$\mathcal{R}(\beta)$ regularization term

Inverse Problem

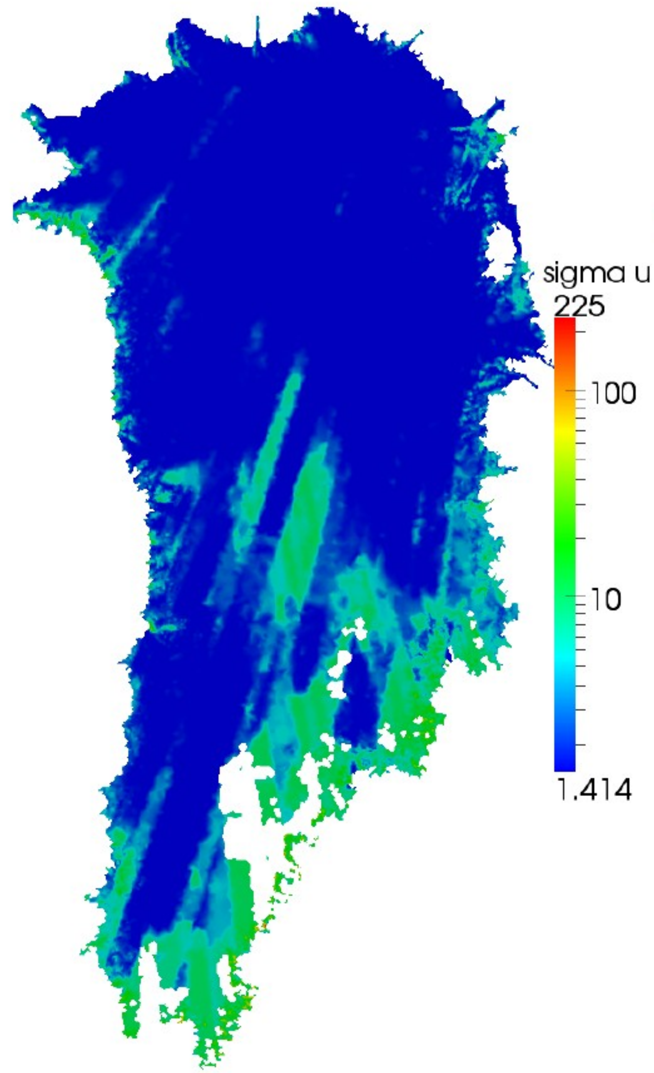
Estimation of ice-sheet initial state of Greenland ice sheet

Grid and RMS of velocity and errors associated with velocity and thickness observations

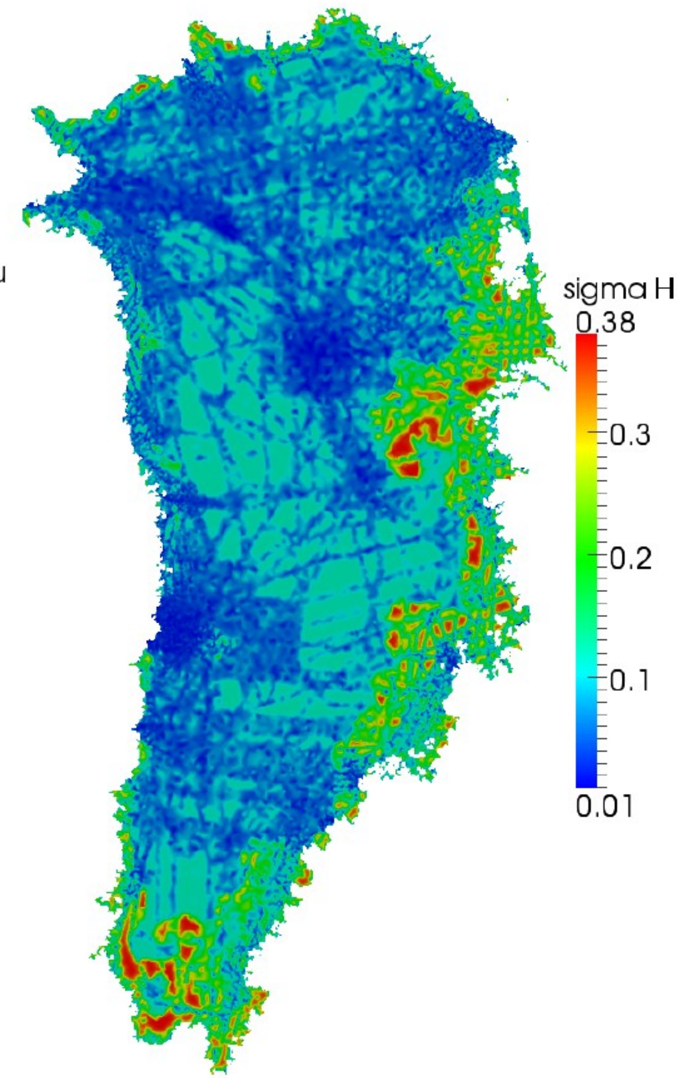
Grid



Velocity RMS (m/yr)



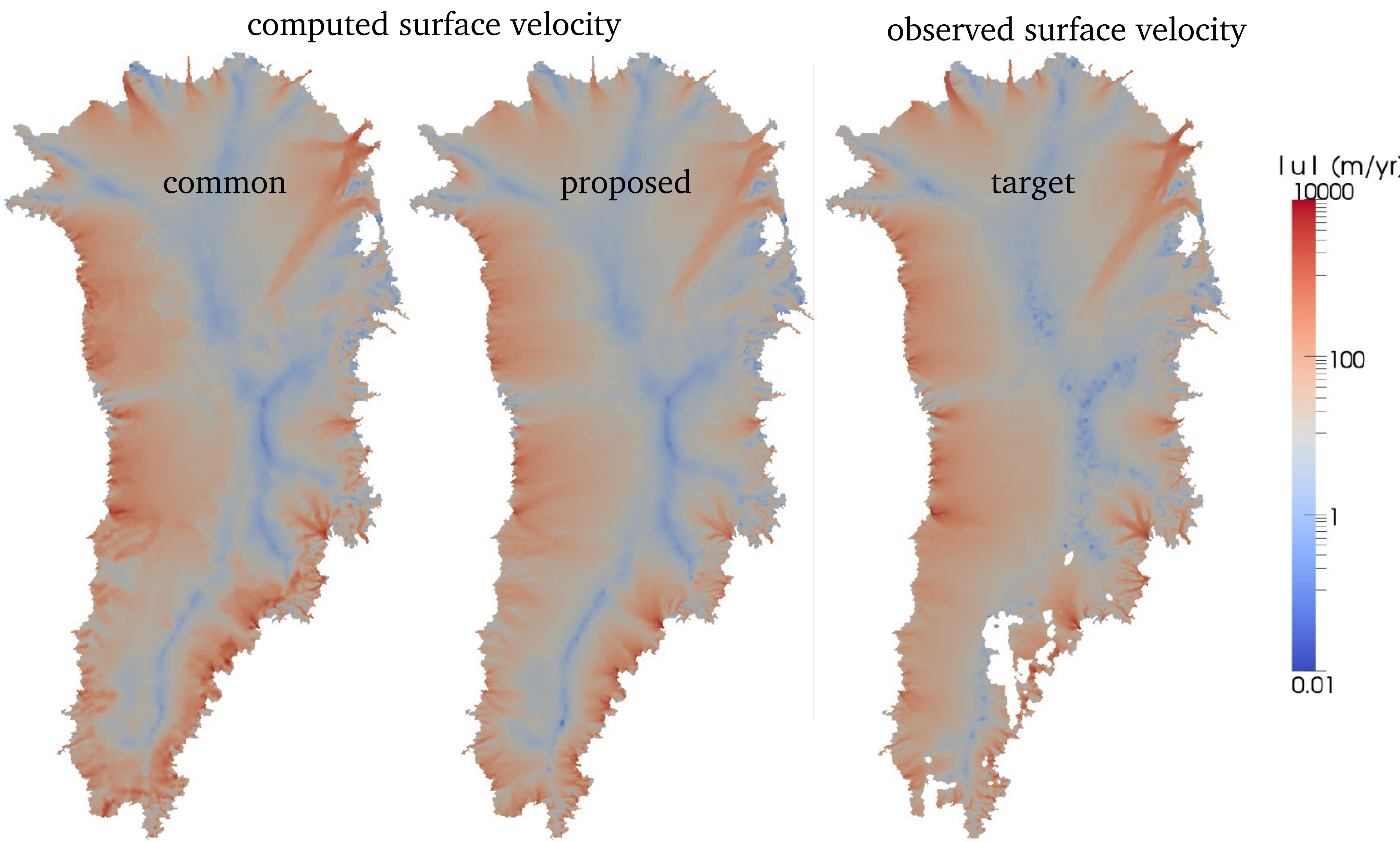
Thickness RMS (km)



Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

Inversion results: surface velocities



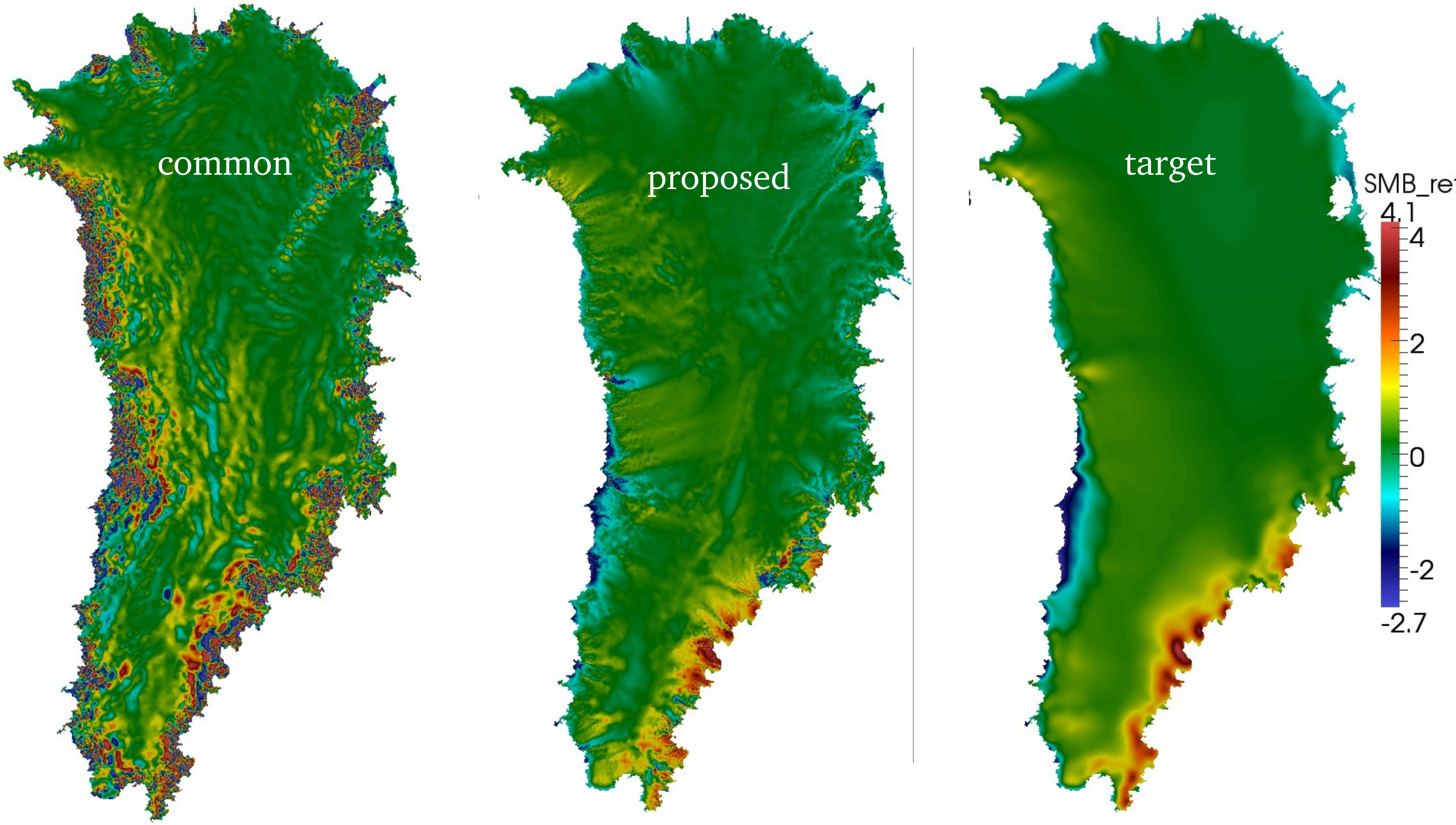
Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

Inversion results: surface mass balance (SMB)

SMB needed for equilibrium

SMB from climate model



Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

Estimated beta and change in topography.

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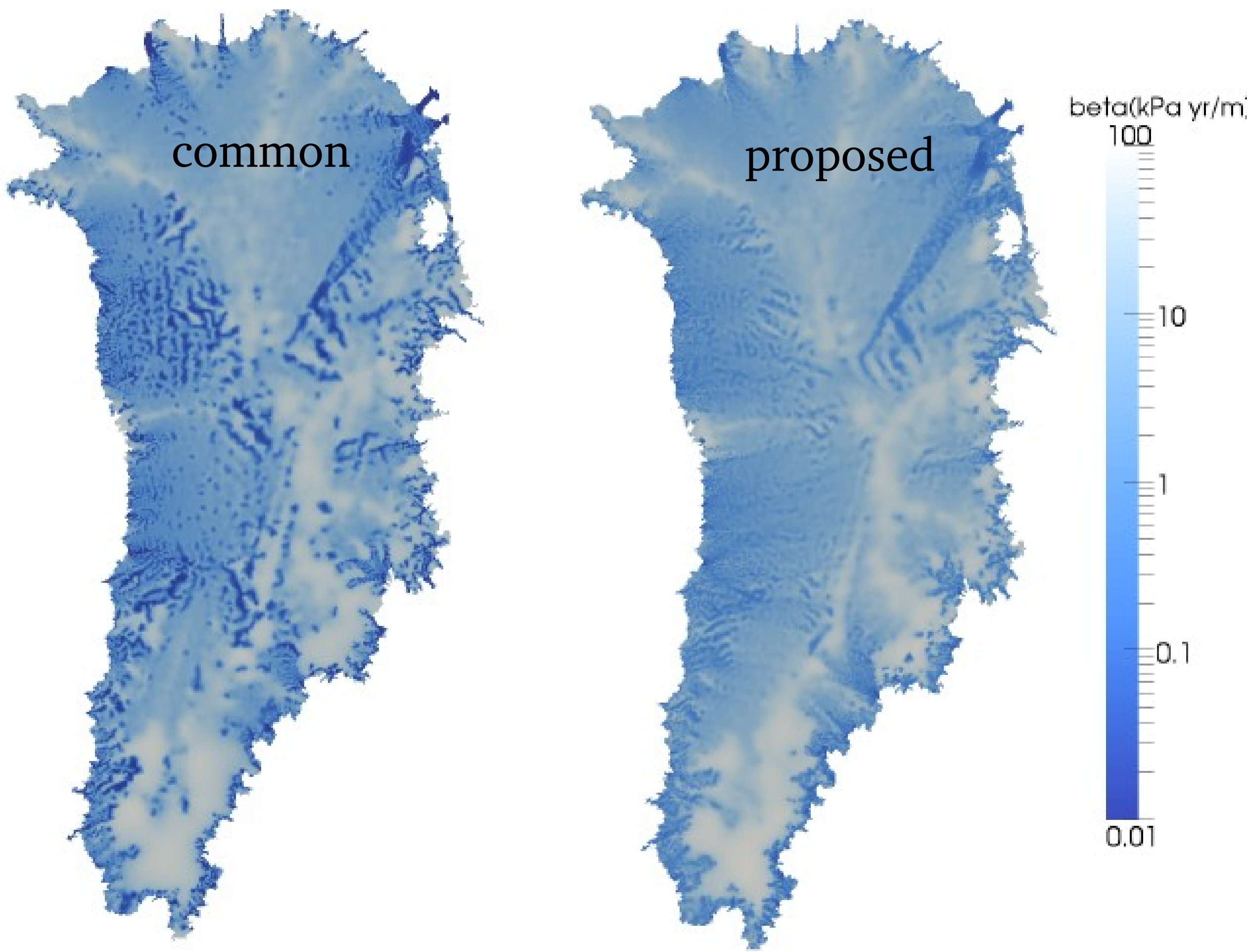
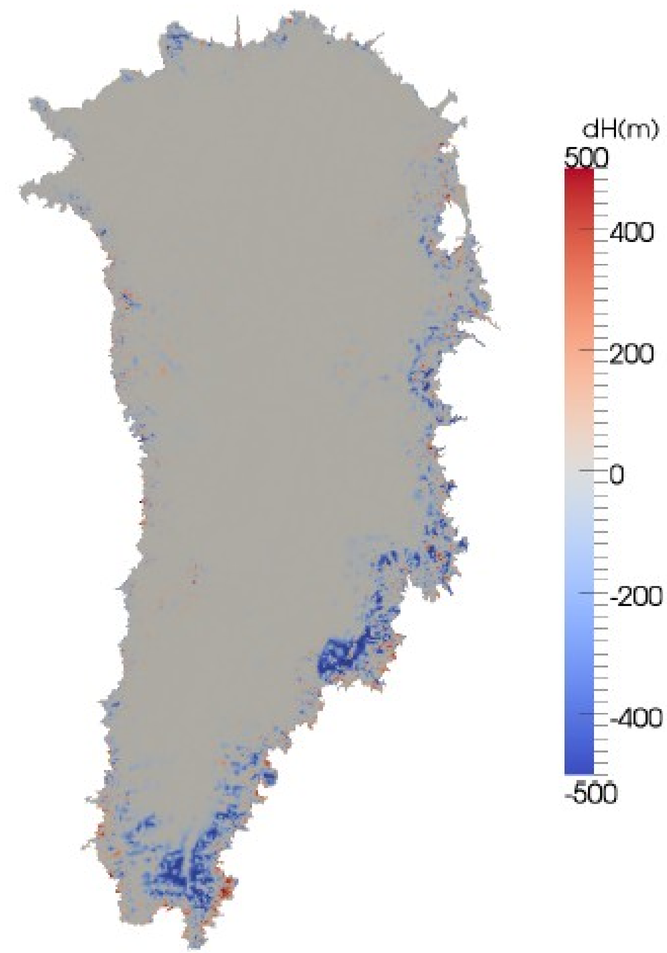
recovered basal friction



difference between recovered and observed thickness



Inverse Problem

Estimation of ice-sheet initial state

Algorithm and Software tools used

Algorithm	Software Tools
Basal non-uniform triangular mesh	*Triangle*
Linear Finite Elements on tetrahedra	*LifeV*
Quasi-Newton optimization (L-BFGS)	**Rol**
Nonlinear solver (Newton method)	**NOX**
Krylov Linear Solvers	**AztecOO/IfPack**

Trilinos

Details:
Regularization terms: Tikhonov.
L-BFGS initialized with Hessian of the regularization terms.

$$\left(\frac{1}{2}\beta^T L\beta \quad \rightarrow \quad L\right)$$

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Inverse Problem

Estimation of ice-sheet initial state

PDE-constraint optimization problem: gradient computation

$$\begin{aligned} \text{Find } (\beta) \text{ that minimize} \quad & \mathcal{J}(\beta, \mathbf{u}) \\ \text{subject to} \quad & \mathcal{F}(\mathbf{u}, \beta) = 0. \end{aligned}$$

flow model

How to compute **total derivatives** of the fuctional w.r.t. the parameters?

$\boxed{\text{Solve State System}}$ $\mathcal{F}(\mathbf{u}, \beta) = 0$

$\boxed{\text{Solve Adjoint System}}$ $\langle \mathcal{F}^*_{\mathbf{u}}(\boldsymbol{\lambda}), \, \boldsymbol{\delta}_{\mathbf{u}} \rangle = \mathcal{J}_{\mathbf{u}}(\boldsymbol{\delta}_{\mathbf{u}}), \quad \forall \boldsymbol{\delta}_{\mathbf{u}}$

$\boxed{\text{Total derivative}}$ $\mathcal{G}(\delta_\beta) = \mathcal{J}_\beta(\delta_\beta) - \langle \boldsymbol{\lambda}, \mathcal{F}_\beta(\delta_\beta) \rangle$

$\boxed{\text{Derivative w.r.t. } \beta}$

$$\mathcal{G}_1(\delta_\beta) = \alpha_\beta \int_\Sigma \nabla \beta \cdot \nabla \delta_\beta \, ds - \int_\Sigma \delta_\beta \, \mathbf{u} \cdot \boldsymbol{\lambda} \, ds$$

Porting the inversion to Albany-FELIX
(w/ E. Phipps, A. Salinger, D. Ridzal and D. Kouri)

Why?
- to exploit Automatic Differentiation for computing derivatives
- to exploit Albany/Trilinos ecosystem (e.g. for UQ capabilities using Dakota)
- to extend Albany adjoint/inversion capabilities,
- to use in-house software (better maintainability)

Albany Development:
- implement distributed parameters, i.e. fields defined on the mesh or on parts of it.
- implement routines for computing derivatives of *residual* and *responses* w.r.t. the distributed parameters.

Trilinos Development:
- couple Piro to ROL using Thyra implementation of ROL::Vector and ROL::Objective. ROL needs reduced gradient and objective functional.

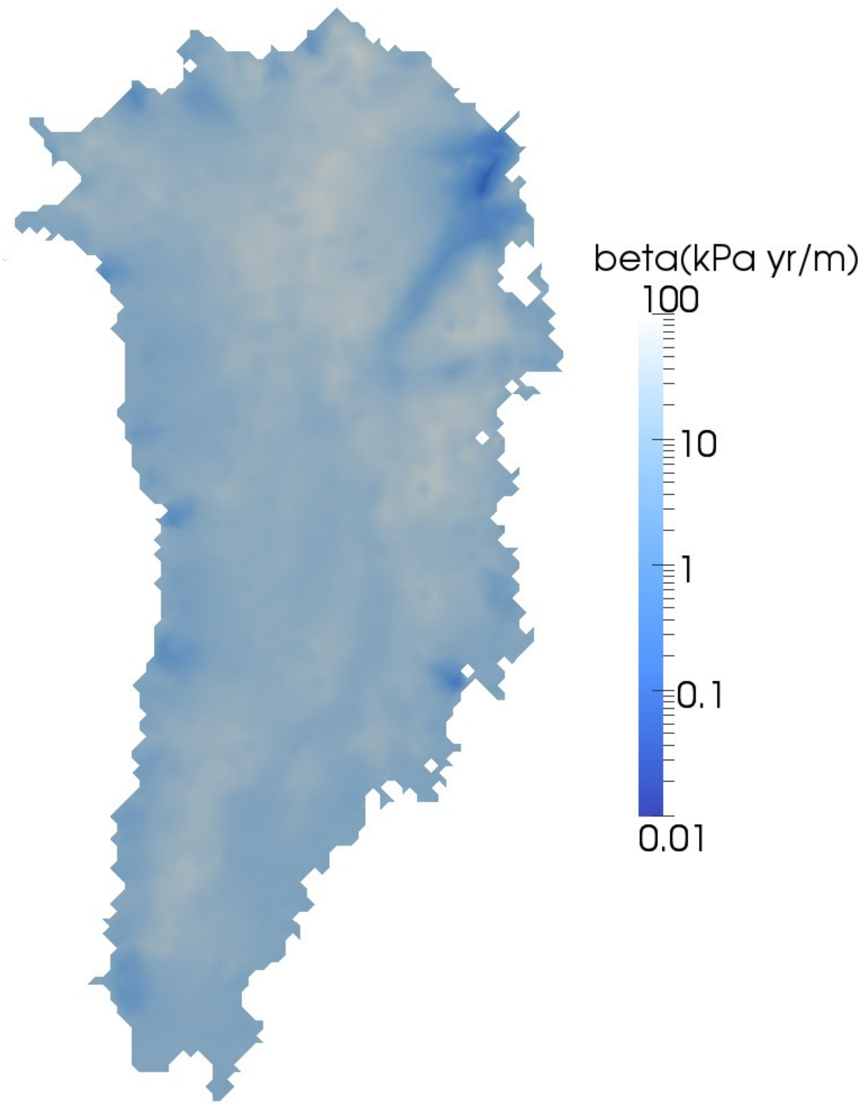
$$\mathcal{G} = \mathcal{J}_\beta - \mathcal{F}_\beta^T \boldsymbol{\lambda}$$

Matrix-free matrix vector product

$\mathcal{G}$: reduced gradient
$\mathcal{J}$: response or objective function
$\mathcal{F}$: residual
β: (distributed) parameter

Preliminary result using Albany-Piro-ROL

recovered basal friction



Objective functional:

$$\mathcal{J}(\mathbf{u}(\beta),\ \beta) = \int_\Sigma \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds + \alpha \int_\Sigma |\nabla \beta|^2 \, ds.$$

ROL algorithm:

- *Limited-Memory BFGS*
- *Backtrack line-search*

Inverted 2000 parameters.

TODOs:

- clean/test Piro-ROL interface
- add bound-constraints

- implement Hessian computation in order to use Newton methods and for UQ

- invert for shape parameters (H)

Trilinos packages used in this calculation:

Nonlinear analysis

- Piro
- ROL
- LOCA
- NOX

Vectors / tools

- Teuchos
- Epetra
- EpetraExt
- Thyra

Physics / discretization

- Phalanx
- Sacado
- Intrepid
- Shards

Linear solvers

- Stratimikos
- Belos/AztecOO
- Ifpack

Mesh

- Zoltan
- STK::Mesh
- STK::IO
- Seacas::Exodus
- Seacas::IOSS

(thanks Andy)