LOCA: Library of Continuation Algorithms

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LOCA: Library of Continuation Algorithms

Application code provides:

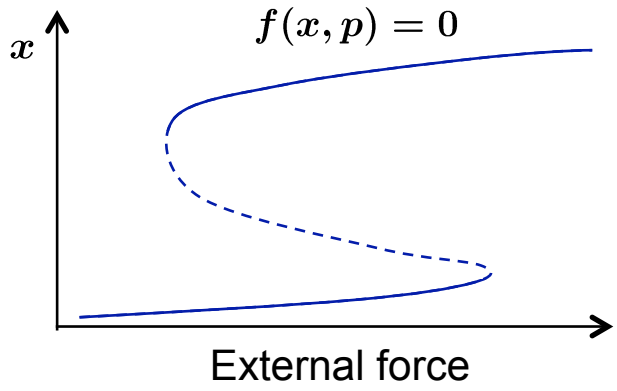
- Nonlinear steady-state residual and Jacobian fill:

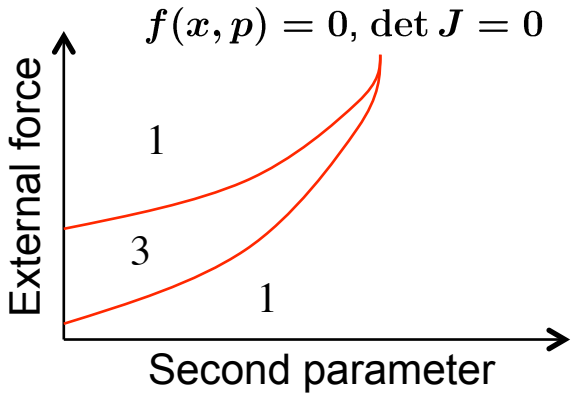
$$f(x,p) = \text{internal} - \text{external force}, \quad J = \frac{\partial f}{\partial x}$$

- Newton-like linear solves: $J\Delta x = -f$

LOCA provides:

- Parameter Continuation: Tracks a family of steady state solutions with parameter
- Linear Stability Analysis: Calculates leading eigenvalues via Anasazi
- Bifurcation Tracking: Locates neutral stability point (x,p) and tracks as a function of a second parameter





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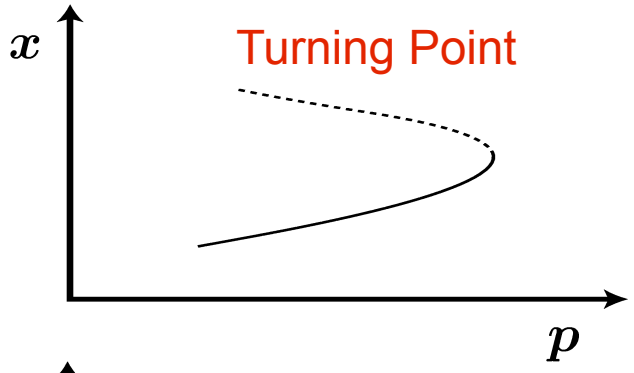
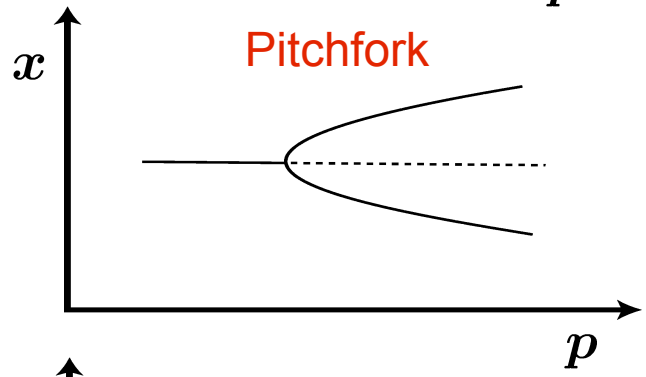
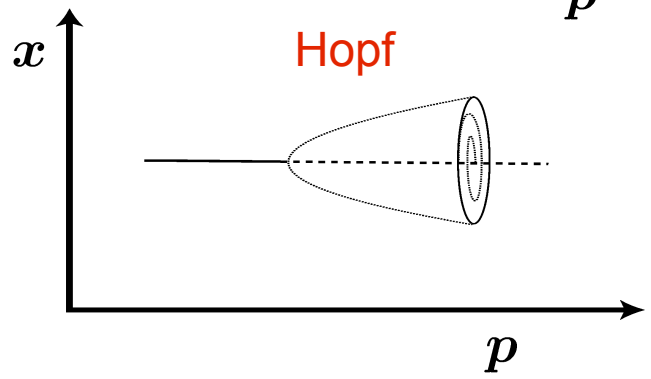
Codimension 1 Bifurcations



- Combustion
- Buckling of an Arch



- Buckling of a Beam
- Pattern formation
- Cell differentiation (morphogenesis)



- Vortex Shedding
- Predator-Prey models
- Flutter
- El Niño

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LOCA Builds Algorithms From NOX's Nonlinear Solver Interface

Application Interface	Matrix Operations on v	Required for
Compute Residual $f(x)$ Compute Jacobian J	Apply Jacobian $J\mathrm{v}$ Apply Jacobian Inverse $J^{-1}\mathrm{v}$	NOX Nonlinear Solver
Set Parameter: p		Parameter Continuation Turning Point Tracking Pitchfork Tracking
Compute Mass Matrix M	Apply Shifted Matrix Apply Shifted Matrix Inverse $(J-\sigma M)\mathrm{v}, \quad (J-\sigma M)^{-1}\mathrm{v}$	Eigensolver
	Apply Complex Matrix Apply Complex Matrix Inverse $(J-i\sigma M)\mathrm{v}, \quad (J-i\sigma M)^{-1}\mathrm{v}$	Hopf Tracking
	Bordered Matrix Inverse $\begin{bmatrix} J & a^T \\ b & c \end{bmatrix}^{-1} \begin{bmatrix} V \\ W \end{bmatrix}$	Minimally Augmented Bifurcations Better arclength implementation

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Ready-Made Implementation for Epetra and Thyra Linear Algebra

Application Interface	Matrix Operations on v	Required for
Compute Residual $f(x)$ Compute Jacobian J	Apply Jacobian $J\mathrm{v}$ Apply Jacobian Inverse $J^{-1}\mathrm{v}$	Nonlinear Solver
Set Parameter: p		Parameter Continuation Turning Point Tracking Pitchfork Tracking
Compute Mass Matrix M	Apply Shifted Matrix Apply Shifted Matrix Inverse $(J-\sigma M)\mathrm{v}, \quad (J-\sigma M)^{-1}\mathrm{v}$	Eigensolver
	Apply Complex Matrix Apply Complex Matrix Inverse $(J-i\sigma M)\mathrm{v}, \quad (J-i\sigma M)^{-1}\mathrm{v}$	Hopf Tracking
	Bordered Matrix Inverse $\begin{bmatrix} J & a^T \\ b & c \end{bmatrix}^{-1} \begin{bmatrix} V \\ W \end{bmatrix}$	Minimally Augmented Turning Point, Pitchfork Better arclength implementation

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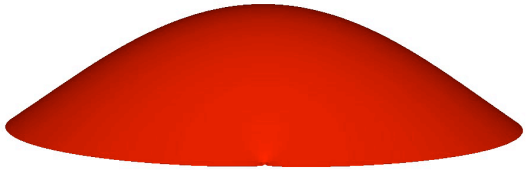
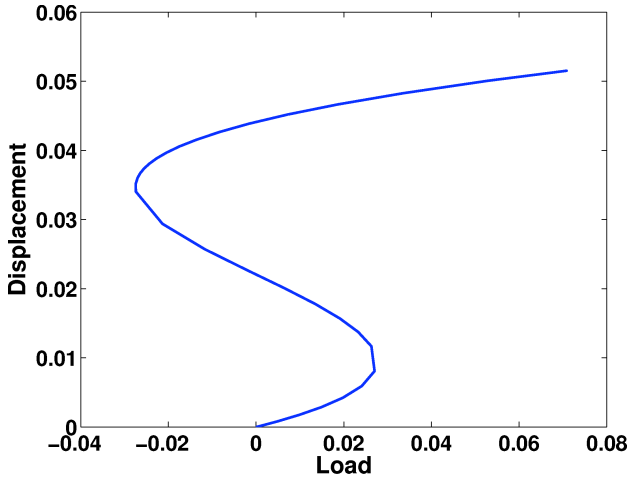
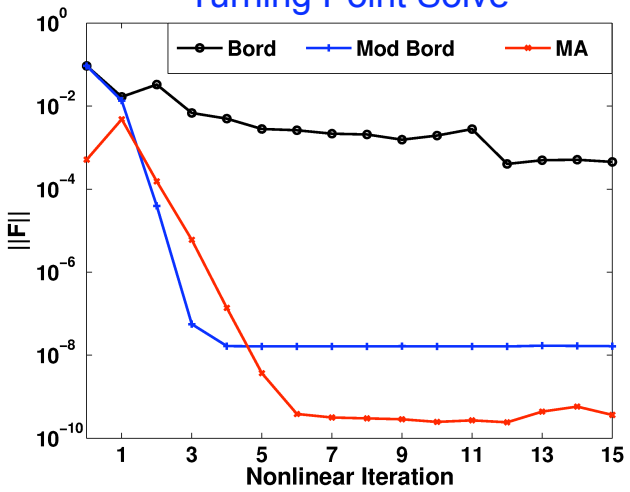
Snap-through Buckling of a Symmetric Cap
(Salinas/FEI, 200K unknowns, 16 procs)



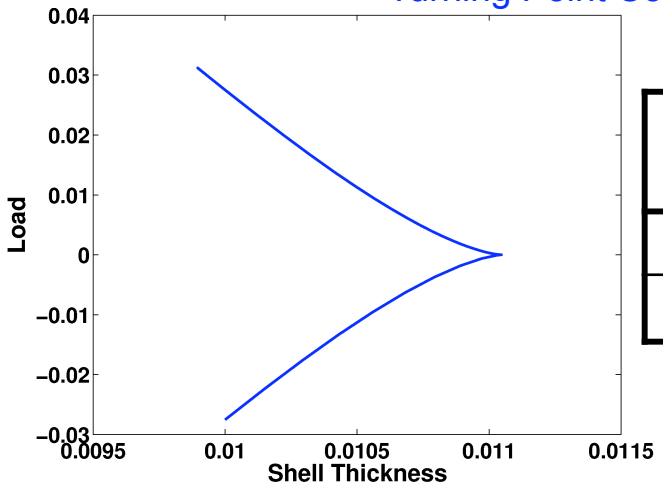
Load/Deflection Curve



Turning Point Solve



Turning Point Continuation



Method	Total Time (hrs)
Mod. Bordering	4.2
Min. Augmented	1.2

- Provides robust buckling info almost 4 times faster than previous best method
- Increases scalability

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Summary of LOCA Capabilities

- LOCA provide robust, scalable algorithms for
  - Parameter continuation
  - Linear stability analysis
  - Bifurcation analysis
  - Periodic orbit tracking
- LOCA builds on NOX's application code interface
  - Most advanced capabilities using Epetra data structures
- LOCA has been successfully leveraged in many research projects
  - Highly encourage external collaborations
- LOCA is in maintenance mode, but new features can always be added if requested
  - Contact Eric Phipps (etphipp@sandia.gov) or Andy Salinger (agsalin@sandia.gov)

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