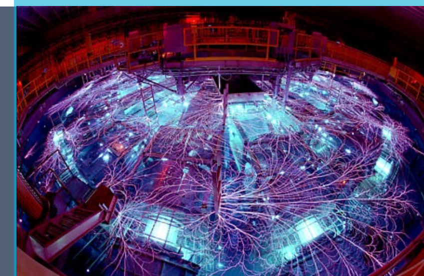
Recent Fusion-Related Results in Drekar



Edward Phillips

Trilinos User-Developer Group Meeting 2023

Fusion Energy Sciences

Advances in Computation Mechanics, Austin, Texas, 10/22/2023

SAND#2023-11657C

Collaborators

Jesus Bonilla and Xian-Zhu Tang, Los Alamos National Lab

Peter Ohm, RIKEN, Japan

John Shadid, Michael Crockatt, Roger Pawlowski,
Ray Tuminaro, Jonathan Hu, Sandia National Labs

Three useful resources for fusion energy for students:

Contemporary Physics Education Project https://www.cpepphysics.org/

Princeton Plasma Physics Laboratory
2021 Introduction to Fusion Energy and Plasma Physics Course
https://suli.pppl.gov/2021/course/

Progress toward fusion energy breakeven and gain as measured against the Lawson criterion
Wurzel, Hsu, Physics of Plasmas 29, 062103 (2022)

Overview of Drekar

Drekar is a finite element based multiphysics code built using Trilinos components with a strong emphasis on implicit time integration methods and linear solvers

Drekar

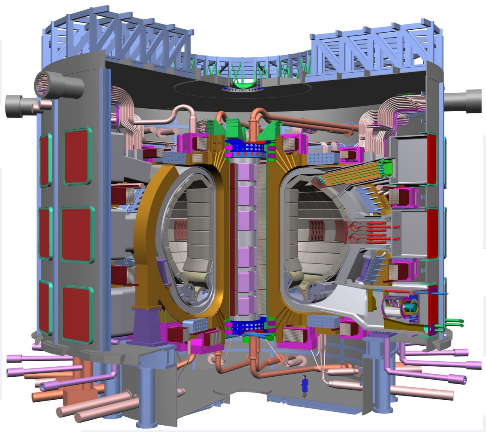
Panzer

Physics	Discretization + Algorithms
Advection-Diffusion-Reaction	Finite Elements (nodal, edge, face, etc.) Intrepid2
Fluid Dynamics	Stabilization (VMS, AFC, etc.)
Electromagnetics	Time Integration (implicit, explicit, IMEX) Tempus
Solids	Nonlinear Solvers NOX
Magnetohydrodynamics	Linear Solvers Belos
Generalized Ohm's Law	Preconditioners Teko, MueLu, Ifpack2
Multifluid Plasma	Adjoints + Optimization Sacado

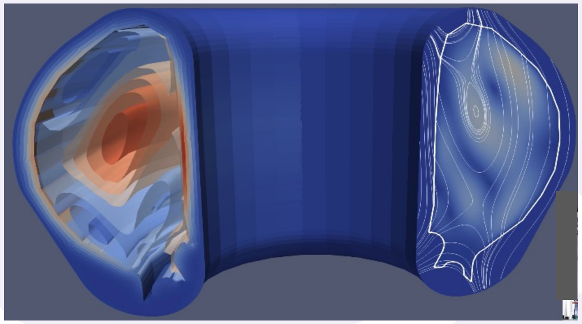
Phalanx

Motivation

- ITER Fusion conditions:
  - Temperature of > 100M deg K (> 6x Sun core temp.).
  - Pulse times of $\mathcal{O}(100) - \mathcal{O}(1000)$ sec. are desired. (energy confinement times of $\mathcal{O}(1) - \mathcal{O}(10)\ sec.$).
- Disruptions can cause
  - Loss of vertical positioning control.
  - Huge thermal energy deposition to the walls, and discharge of very large electrical currents to surface.
  - Huge forces can be generated in first wall & vacuum vessel structures.
- ITER can sustain only a limited number of significant disruptions, $\mathcal{O}(1 - 5)$.
- Understanding and controlling instabilities/disruptions in plasma confinement is critical



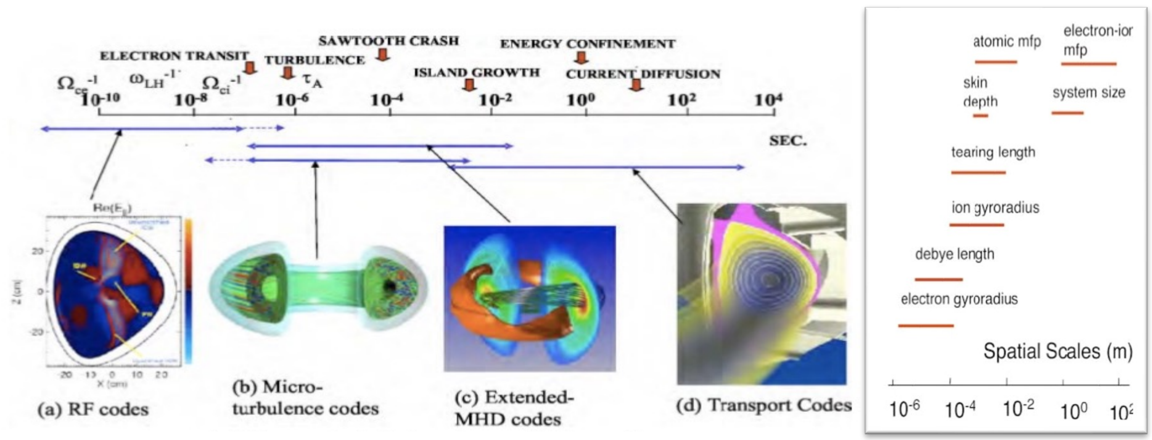
ITER Tokamak



Vertical Displacement Event (VDE) in ITER Tokamak Plasma and Wall Geometry (Drekar sim.)

Goal

- Analyze macroscopic plasma **3D** behavior/instabilities under ITER conditions.

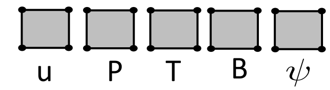


DOE Office of Science ASCR/OFES Reports: Fusion Simulation Project Workshop Report, 2007, Integrated System Modeling Workshop 2015

- **Implicit** time integration (overstep fastest time scales/CFL on refined meshes).
- **Stabilized FE formulation** (resolution, higher-order accuracy, enforcing constraints)

  (currently) Single-fluid compressible visco-resistive MHD.
- **Newton-Krylov + multiphysics block preconditioners (Highly scalable, efficient and robust solvers)**

A Reduced length-scale/time scale representation; Basic single fluid Resistive MHD [e.g. 3D H(grad) Variational Multiscale (VMS) Stabilized FE]



<u>Resistive MHD Model in Conservative Form</u>

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \nabla\cdot[\rho\mathbf{v}\otimes\mathbf{v} + P\mathbf{I} - \mathbf{\Pi}] - \mathbf{J}\times\mathbf{B} = 0$$

$$\mathbf{\Pi} = \frac{2}{3}\mu(\nabla\cdot\mathbf{u})\mathbf{I} + \mu[\nabla\mathbf{u} + \nabla\mathbf{u}^T]$$

Sound wave off diagonal coupling, and nearly incompressible flow limit ($\nabla\cdot\mathbf{v}\approx 0$)

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

Alfven wave strong off diagonal coupling

$$\frac{\partial(\rho e)}{\partial t} + \nabla\cdot[\rho\mathbf{v}e + \mathbf{q}] - \mathbf{T}:\nabla\mathbf{v} - \eta\left\|\frac{1}{\mu_0}\nabla\times\mathbf{B}\right\|^2 = 0$$

$$\frac{\partial\mathbf{B}}{\partial t} + \nabla\cdot\left[\mathbf{B}\otimes\mathbf{v} - \mathbf{v}\otimes\mathbf{B} - \frac{\eta}{\mu_0}\left(\nabla\mathbf{B} - (\nabla\mathbf{B})^T\right) + \psi\mathbf{I}\right] = \mathbf{0}$$

$$\nabla\cdot\mathbf{B} = 0$$

Elliptic constraint – infinite wave speed

<u>MHD Wave speeds</u>

$\|\mathbf{u}\|, \|\mathbf{u}\|\pm c_s, \|\mathbf{u}\|\pm c_a, \|\mathbf{u}\|\pm c_f, \pm c_h$

Here c_h is ∞ for elliptic divergence cleaning

Heterogeneous Multiphysics: 3D implicit unstructured FE visco-resistive MHD in ITER geometry

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot [\rho \mathbf{v} \otimes \mathbf{v} - (\mathbf{T} + \mathbf{T}_M)] + 2\rho\Omega \times \mathbf{v} - \rho \mathbf{g} = \mathbf{0}$$

$$\mathbf{T} = -\left(P + \frac{2}{3}\mu(\nabla \bullet \mathbf{u})\right)\mathbf{I} + \mu[\nabla \mathbf{u} + \nabla \mathbf{u}^T]$$

$$\mathbf{T_M} = \frac{1}{\mu_0}\mathbf{B} \otimes \mathbf{B} - \frac{1}{2\mu_0}\|\mathbf{B}\|^2 \mathbf{I}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot [\rho \mathbf{v} e + \mathbf{q}] - \mathbf{T} : \nabla \mathbf{v} - \eta \left\| \frac{1}{\mu_0} \nabla \times \mathbf{B} \right\|^2 = 0$$

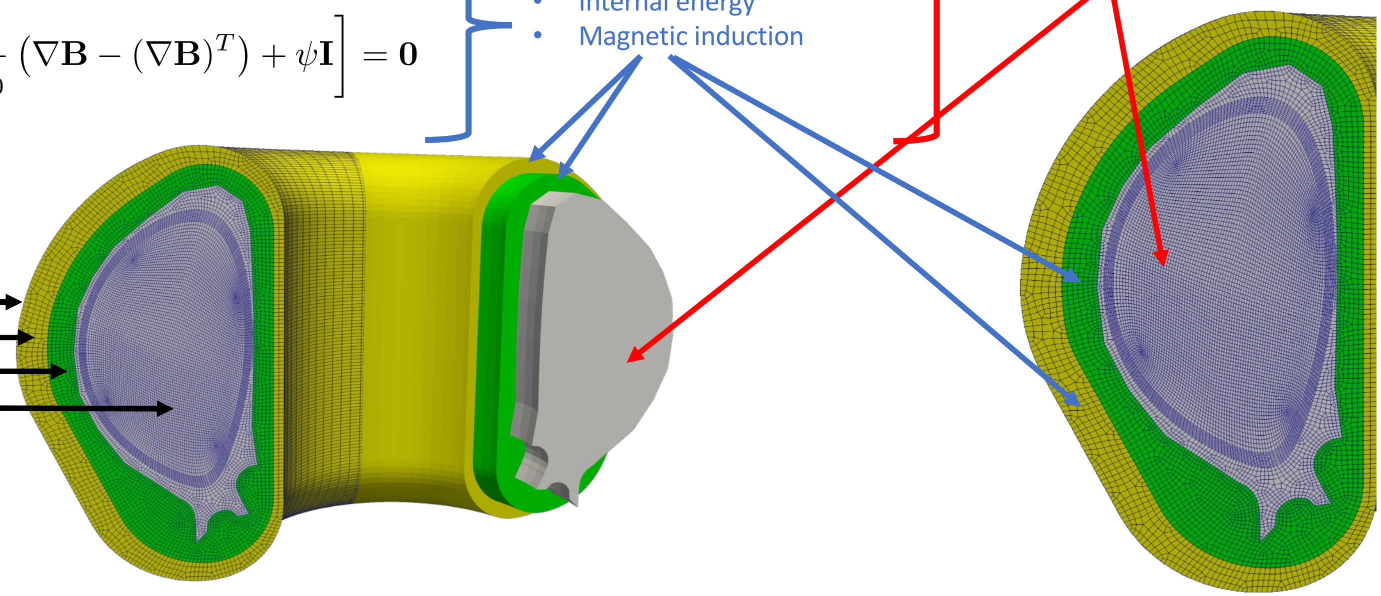
$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot \left[\mathbf{B} \otimes \mathbf{v} - \mathbf{v} \otimes \mathbf{B} - \frac{\eta}{\mu_0}\left(\nabla \mathbf{B} - (\nabla \mathbf{B})^T\right) + \psi \mathbf{I}\right] = \mathbf{0}$$

$$\nabla \cdot \mathbf{B} = 0$$

Plasma region: visco-resistive MHD
- Momentum
- Mass
- Internal energy
- Magnetic induction

Solid walls and vacuum regions
- Internal energy
- Magnetic induction

Heterogeneous Physics Regions
- Outer perfect conducting wall
- Vacuum region
- Solid 1st wall
- Plasma region



Discretization

Finite element discretization (Galerkin terms)

Find $\mathbf{U} \doteq [\rho, \mathbf{m}, T, \mathbf{B}, \psi]^T \in \mathcal{U}$ such that $\rho = \overline{\rho}$ on Γ_D^ρ, $\mathbf{m} = \overline{\mathbf{m}}$ on $\Gamma_D^\mathbf{m}$, $T = \overline{T}$ on Γ_D^T, $\mathbf{B} = \overline{\mathbf{B}}$ on $\Gamma_D^\mathbf{B}$, $\psi = \overline{\psi}$ on Γ_D^ψ, and

$$\mathcal{A}(\mathbf{W}, \mathbf{U}) = \mathcal{F}(\mathbf{W}) \quad \forall\ \mathbf{W} \doteq [q, \mathbf{w}, \theta, \mathbf{C}, s]^T \in \mathcal{V},$$

where

$$\begin{aligned}\mathcal{A}(\mathbf{W}, \mathbf{U}) \doteq\ & (q, \partial_t \rho) - (\nabla q, \rho \mathbf{u}) \\ & + (\mathbf{w}, \partial_t \rho \mathbf{u}) - \langle \nabla \mathbf{w}, \rho \mathbf{u} \otimes \mathbf{u} \rangle - (\nabla \cdot \mathbf{w}, p + \tfrac{2}{3Re}(\nabla \cdot \mathbf{u})) + \langle \nabla \mathbf{w}, \tfrac{1}{Re}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \rangle - (\mathbf{w}, \mathbf{j} \times \mathbf{B}) \\ & + (\theta, \partial_t T) + (\theta, \mathbf{u} \cdot \nabla T) + \frac{2}{3}(\theta, T(\nabla \cdot \mathbf{u})) + (\nabla \theta, \mathbf{q}) \\ & + (\mathbf{C}, \partial_t \mathbf{B}) - \langle \nabla \mathbf{C}, \mathbf{u} \otimes \mathbf{B} - \mathbf{B} \otimes \mathbf{u} \rangle + \left\langle \nabla \mathbf{C}, \tfrac{1}{S} \left(\nabla \mathbf{B} - (\nabla \mathbf{B})^T \right) \right\rangle - (\nabla \cdot \mathbf{C}, \psi) \\ & + (s, \nabla \cdot \mathbf{B}),\end{aligned}$$

$$(a, b) = \int_\Omega ab\ d\Omega$$

$$(\mathbf{a}, \mathbf{b}) = \int_\Omega \mathbf{a} \cdot \mathbf{b}\ d\Omega$$

$$\langle \mathbf{A}, \mathbf{B} \rangle = \int_\Omega \mathbf{A} : \mathbf{B}\ d\Omega$$

Deficiencies of Galerkin Weak Form:

- Equal-order interpolation have stability problems for saddle point prbs. (LBB condition, see .e.g. Gunzburger 1989)
  - Induction – div B = 0; Lagrange multiplier coupling $(\mathbf{B}, \psi)$
  - Strong guide field (large **B**) produces an incompressible flow limit type response and a saddle point like structure (e.g. Stokes-like behavior for $(\rho\mathbf{u}, \rho)$)
- Strong convective transport and large unresolved gradients can produce unphysical spatial oscillations (internal / boundary layers).
- For unresolved high-wavenumber signals aliasing of energy into lower-wavenumber resolved components

Brief Outline Following Variational Multiscale (VMS) Approach
VMS: T.J.R Hughes et. al.; & VMS MHD: Codina et. al., JS et. al.

Upwinding and saddle point stabilization

Let us split the solution and test spaces in resolved and unresolved scales, i.e., $\mathcal{U} = \mathcal{U}_h + \mathcal{U}'$ and $\mathcal{V} = \mathcal{V}_h + \mathcal{V}'$, thus we have

$$\mathcal{A}(\mathbf{W}_h, \mathbf{U}_h + \mathbf{U}') = \mathcal{F}(\mathbf{W}_h) \quad \forall\, \mathbf{W}_h \in \mathcal{V}_h$$

$$\cancel{\mathcal{A}(\mathbf{W}', \mathbf{U}_h + \mathbf{U}') = \mathcal{F}(\mathbf{W}') \quad \forall\, \mathbf{W}' \in \mathcal{V}'} \longrightarrow \mathbf{U}' \text{ not resolved, modeled by } \mathbf{U}' \approx -\boldsymbol{\tau}\mathbf{P}\mathcal{R}(\mathbf{U}^h)$$

I.e. sub-grid / unresolved scales driven by residual resolved scales of strong from PDEs, variationally consistent

VMS + additional optional DCO terms are included for enhanced stability

$$\begin{aligned}
\mathcal{A}(\mathbf{W}_h, \mathbf{U}_h + \mathbf{U}') = \mathcal{A}(\mathbf{W}_h, \mathbf{U}_h) - \sum_{K \in \mathcal{T}_h} \Big(& \left.\begin{aligned}
&(\nabla q_h, \rho_h \mathbf{u}' + \mathbf{u}_h \rho')_K \\
&+ \langle \nabla \mathbf{w}_h, \rho \mathbf{u}' \otimes \mathbf{u}_h + \rho' \mathbf{u}_h \otimes \mathbf{u}_h \rangle_K + (\nabla \cdot \mathbf{w}_h, p')_K \\
&+ (\nabla \theta_h, \mathbf{u}_h T')_K \\
&+ \langle \nabla \mathbf{C}_h, \mathbf{u}_h \otimes \mathbf{B}' - \mathbf{B}' \otimes \mathbf{u}_h \rangle_K + (\nabla \cdot \mathbf{C}_h, \psi')_K \\
&+ (\nabla s_h, \mathbf{B}')_K
\end{aligned}\right\} \text{VMS} \\
& \left.+ (\nabla q_h, \nu_\rho^K \nabla \rho_h)_K \right\} \text{DCO LLF [density]} \\
& \left.+ (\nabla \mathbf{w}_h, \tfrac{1}{2} \nu_{\mathbf{m}}^K (\nabla \mathbf{u}_h + (\nabla \mathbf{u}_h)^T))_K + \langle \nabla \mathbf{w}_h, \nu_\rho^K \nabla \rho_h \otimes \mathbf{u}_h \rangle_K \right\} \text{DCO R(s) [momentum]} \\
& \left.+ \frac{C}{2}(u_A h\, \hat{b} \cdot \nabla \theta, \hat{b} \cdot \nabla T)_K \quad \Big) \right\} \hat{b} \text{ DCO [temperature]}
\end{aligned}$$

First order cG finite elements for $\rho_h, \mathbf{m}_h, \mathbf{B}_h, \psi_h$ and **second order** finite elements T_h.

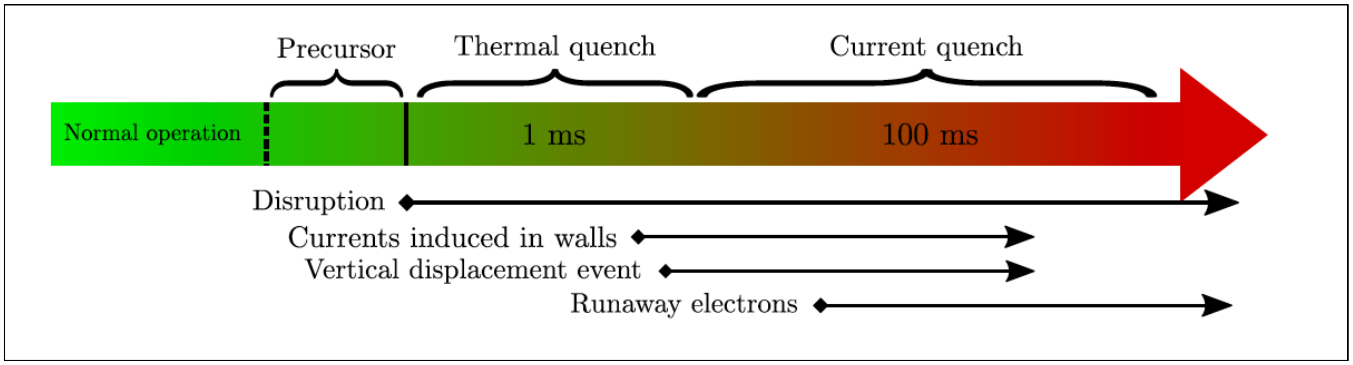
Bonilla, S, Tang, Crockatt, Ohm, Phillips, Pawlowski, Conde, Beznosov, On a Fully-implicit VMS-stabilized FE Formulation for Low Mach Number Compressible Resistive MHD with Application to MCF, Comput. Methods Appl. Mech. Engrg. 2023

S, Pawlowski, Cyr, Tuminaro, Chacon, Weber, Scalable Implicit Incompressible Resistive MHD with Stabilized FE and Fully-coupled Newton-Krylov-AMG, Comput. Methods Appl. Mech. Engrg. 304, 1–25, 2016

Preliminary Results

Vertical displacement events (VDEs) are major disruption events occurring in tokamaks when vertical stability control is lost.

These events cause large currents to flow in the vessel and other adjacent metallic structures.



1. **Cold VDE, Thermal Quench – a fast internal energy loss (i.e. Temperature drop)**
   1. **Initial equilibrium force balance is lost:** $(\mathbf{J} \times \mathbf{B})_{0^+} \neq (\nabla P)_{0^+}$
   2. **Loss of vertical position control**

2. **Temperature drop => resistivity increases => plasma current drops + ohmic to runaway electron current conversion.**

3. **Plasma current drop, new sequence of quasi-steady "equilibriums" => magnetic field rearrangement. i.e. VDE**

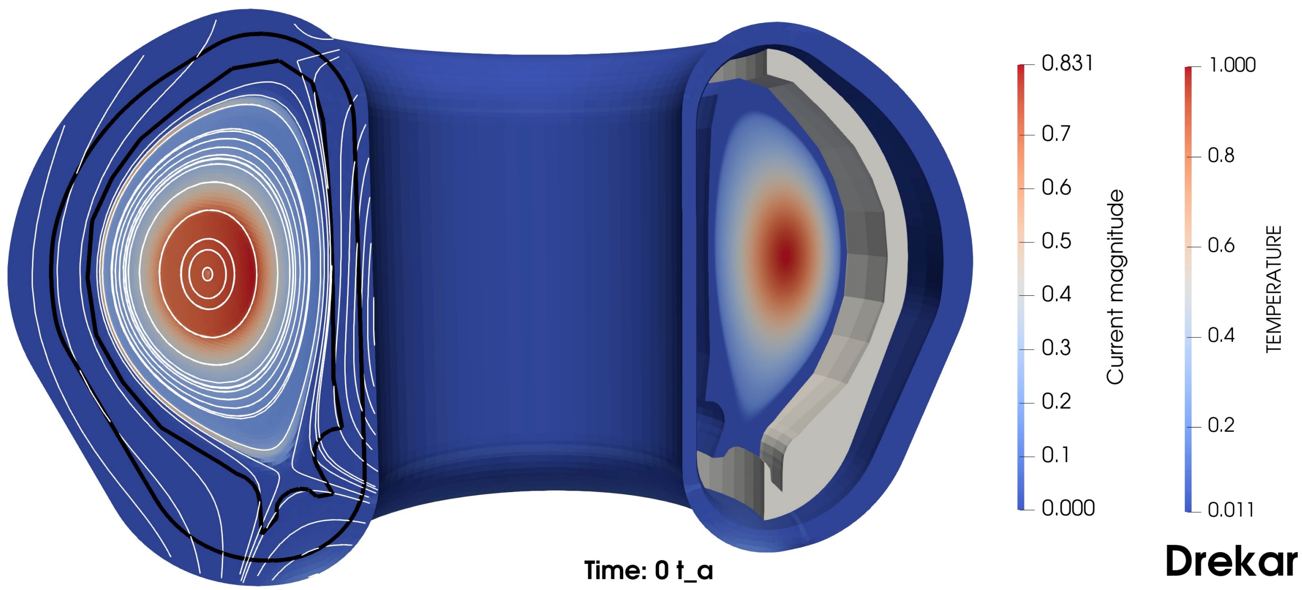
4. **VDE => large halo currents produced in walls and large EM forces**

Vertical Displacement Event (VDE) II

Parameters

- D_2 plasma
- B_0 : $5T$
- T : 40 keV (> 100M K)
- n : 10^{20}
- S_p : 10^4 – 10^7 (Scaling results up to 10^{10})

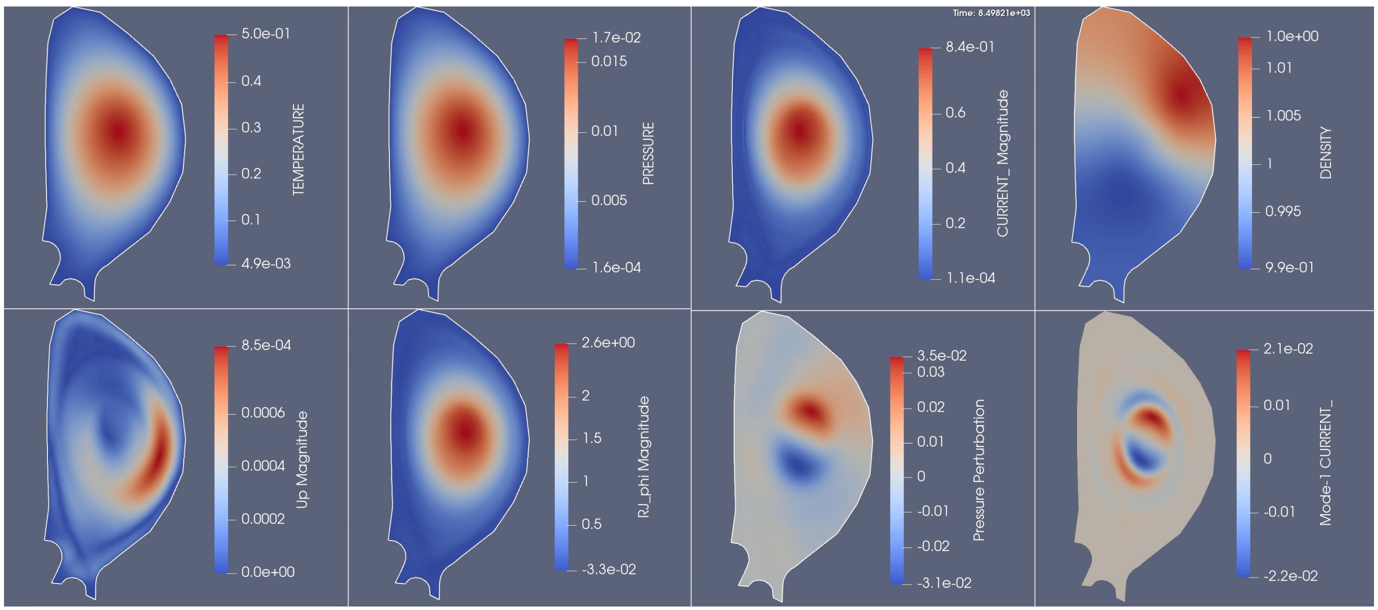
- S_w : 10^4
- S_{vv} : 10^5
- $\kappa_\perp$: 10^{-3} (thermal quench)
- ν : 10^{-2}



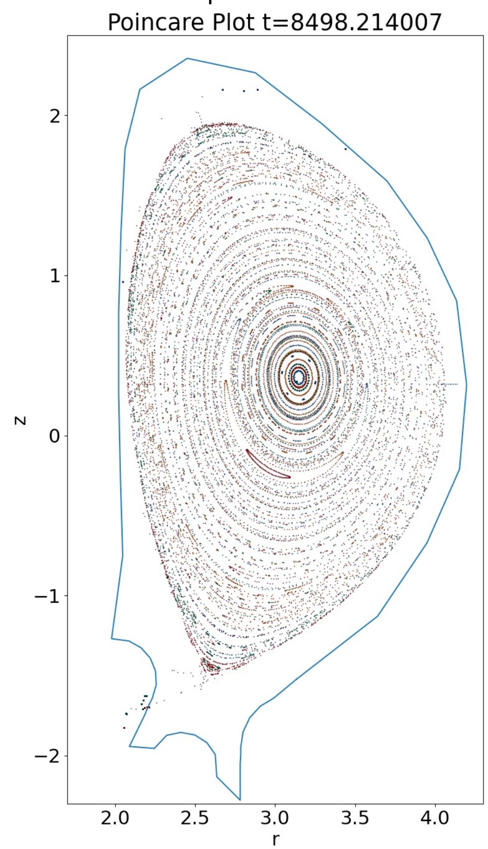
Internal kink

- Initial equilibrium with q profile < 0.9 (very unstable)
- Introduced (1,1) perturbation and let evolve in time
  - (1,1) leads to sawtooth crash with island growth
  - (2,1) is excited leading to stochastic magnetic field
  - breakdown of magnetic surfaces and a disruption.

- B_0 : $5T$
- n : 10^{20}
- Sp : $10^4 - 10^6$
- $\kappa_\perp$: 10^{-5}
- $\kappa_\parallel$: 10^{-3}
- ν : 10^{-5}



Return map of B to Poloidal plane:



Physics-based and Approximate Block Factorizations: Coercing Strongly Coupled Off- Diagonal Physics and/or Disparate Discretizations and Scalable Multigrid to play well together

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial x} \,, \quad \frac{\partial v}{\partial t} = \frac{\partial u}{\partial x}$$

Semi-discrete in time (e.g. BE):
Eliminate variable to parabolic form

$$u^{n+1} = u^n + \Delta t \frac{\partial v^{n+1}}{\partial x} \,, \quad v^{n+1} = v^n + \Delta t \frac{\partial u^{n+1}}{\partial x}$$

$$\frac{u^{n+1} - u^n}{\Delta t} - \Delta t \frac{\partial^2 u^{n+1}}{\partial x^2} = \frac{\partial v^n}{\partial x}$$

The resulting equation

$$(I - \Delta t^2 \mathcal{L}_{xx}) u^{n+1} = \mathcal{F}^n$$

Semi-discrete in time:
Approximate Block Factorizations & Schur-complements:

$$\begin{bmatrix} I & -\Delta t C_x \\ -\Delta t C_x & I \end{bmatrix} \begin{bmatrix} u^{n+1} \\ v^{n+1} \end{bmatrix} = \begin{bmatrix} u^n - \Delta t C_x v^n \\ v^n - \Delta t C_x u^n \end{bmatrix}$$

$$\begin{bmatrix} D_1 & U \\ L & D_2 \end{bmatrix} = \begin{bmatrix} I & U D_2^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} D_1 - U D_2^{-1} L & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} I & 0 \\ D_2^{-1} L & I \end{bmatrix}$$

The Schur complement is then

$$D_1 - U D_2^{-1} L = (I - \Delta t^2 C_x C_x) \approx (I - \Delta t^2 \mathcal{L}_{xx})$$

Recall: This is motivating how we develop preconditioners, not for developing solvers.
The NK method still seeks the solution to the original nonlinear/linear system residual!

[w/ L. Chacon (LANL)]

$$\boxed{\begin{array}{c}[\mathbf{F_{ns}} = [\mathbf{F}_{\rho\mathbf{u}}, \mathbf{F}_{\rho}], \mathbf{F_B}, \mathbf{L_r}, F_T] \\ \text{Block Jacobi}\end{array}}$$

$$\begin{bmatrix} \mathbf{F}_{\mathrm{ns}} & \mathbf{Z} & & C_T \\ \mathbf{Y} & \mathbf{F}_{\mathbf{B}} & \mathcal{B}_{\mathbf{B}}{}^T & \\ \mathbf{C}_{\mathrm{ns}} & \mathcal{B}_{\mathbf{B}} & \mathbf{L}_{\mathbf{r}} & \\ A_T & \mathbf{Z}_T & & \mathrm{F}_{\mathrm{T}} \end{bmatrix}^{-1}$$

<table>
<tr><td colspan="2">$[\mathbf{F}_{\mathbf{ns}} = [\mathbf{F}_{\rho\mathbf{u}}, \mathbf{F}_{\rho}], \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}, F_T]$
Block Jacobi</td></tr>
<tr><td>$[\mathbf{F}_{\mathrm{ns}}, \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}]$
Operator Splitting</td><td>$[\mathrm{F}_{\mathrm{T}}]$
AMG</td></tr>
</table>

$$
\begin{bmatrix}
\begin{bmatrix}
\mathbf{F}_{\mathrm{ns}} & \mathbf{Z} & \\
\mathbf{Y} & \mathbf{F}_{\mathbf{B}} & \mathcal{B}_{\mathbf{B}}{}^{T} \\
\mathbf{C}_{\mathrm{ns}} & \mathcal{B}_{\mathbf{B}} & \mathbf{L}_{\mathbf{r}}
\end{bmatrix}^{-1} & \\
 & [\mathrm{F}_{\mathrm{T}}]^{-1}
\end{bmatrix}
$$

<table>
<tr><td colspan="3">$[\mathbf{F}_{\mathbf{ns}} = [\mathbf{F}_{\rho\mathbf{u}}, \mathbf{F}_{\rho}], \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}, F_T]$
Block Jacobi</td></tr>
<tr><td colspan="2">$[\mathbf{F}_{\mathrm{ns}}, \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}]$
Operator Splitting</td><td>$[\mathrm{F}_{\mathrm{T}}]$
AMG</td></tr>
<tr><td>$[\mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}]$
LU Decomp. + SIMPLEC</td><td>$[\mathbf{F}_{\mathbf{B}}, \mathbf{F}_{\mathrm{ns}}]$
LU Decomp. + SIMPLEC</td><td></td></tr>
</table>

$$\left(\begin{bmatrix} \mathbf{F}_{\mathrm{ns}} & \mathbf{Z} & \\ \mathbf{Y} & \mathbf{F}_{\mathbf{B}} & \\ & & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & & \\ & {\mathbf{F}_{\mathbf{B}}}^{-1} & \\ \mathbf{C}_{\mathrm{ns}} & & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{F}_{\mathbf{B}} & {\mathcal{B}_{\mathbf{B}}}^{T} \\ & \mathcal{B}_{\mathbf{B}} & \mathbf{L}_{\mathbf{r}} \end{bmatrix} \right)^{-1}$$

2x2 critical implicit Stiff Alfven wave coupling

2x2 Saddle point system for (B,r)

<table>
<tr><td colspan="5">$[\mathbf{F_{ns}} = [\mathbf{F}_{\rho\mathbf{u}}, \mathbf{F}_{\rho}], \mathbf{F_B}, \mathbf{L_r}, F_T]$
Block Jacobi</td></tr>
<tr><td colspan="4">$[\mathbf{F}_{\mathrm{ns}}, \mathbf{F_B}, \mathbf{L_r}]$
Operator Splitting</td><td>$[\mathrm{F_T}]$
AMG</td></tr>
<tr><td colspan="2">$[\mathbf{F_B}, \mathbf{L_r}]$
LU Decomp. + SIMPLEC</td><td colspan="2">$[\mathbf{F_B}, \mathbf{F}_{\mathrm{ns}}]$
LU Decomp. + SIMPLEC</td><td></td></tr>
<tr><td>$[\mathbf{F_B}]$
AMG</td><td>$[S_L]$
AMG</td><td>$[S_{mag}]$
AMG</td><td>$[\mathbf{F}_{\mathrm{ns}}]$
LU Decomp. + SIMPLEC</td><td></td></tr>
</table>

$$\begin{bmatrix} \mathbf{I} & & \\ & \mathbf{F_B} & \mathcal{B}_{\mathbf{B}}{}^T \\ & & S_L \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{F_B} & \\ & -\mathcal{B}_{\mathbf{B}} & \mathbf{I} \end{bmatrix}, \qquad \begin{bmatrix} \mathbf{F}_{\mathrm{ns}} & \mathbf{Z} & \\ & S_{mag} & \\ & & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} & & \\ -\mathbf{Y}\,\mathbf{F}_{\mathrm{ns}}{}^{-1} & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix}$$

$$S_L := \mathbf{L_r} - \mathcal{B}_{\mathbf{B}}(\mathrm{absrowsum}(\mathbf{F_B}))^{-1}\,\mathcal{B}_{\mathbf{B}}{}^T, \qquad S_{mag} := \mathbf{F_B} - \mathbf{Y}(\mathrm{absrowsum}(\mathbf{F}_{\mathrm{ns}}))^{-1}\,\mathbf{Z}$$

$$\mathbf{F}_{ns} = \begin{bmatrix} \mathbf{F}_{\rho u} & \mathbf{B}^T_{\rho u,\rho} \\ \mathbf{B}_{\rho,\rho u} & \mathbf{F}_{\rho} \end{bmatrix}$$

$[\mathbf{F}_{\mathbf{ns}} = [\mathbf{F}_{\rho\mathbf{u}}, \mathbf{F}_{\rho}], \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}, F_T]$
Block Jacobi

$[\mathbf{F}_{\mathrm{ns}}, \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}]$
Operator Splitting

$[\mathrm{F}_{\mathrm{T}}]$
AMG

$[\mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}]$
LU Decomp. + SIMPLEC

$[\mathbf{F}_{\mathbf{B}}]$
AMG

$[S_L]$
AMG

$[\mathbf{F}_{\mathbf{B}}, \mathbf{F}_{\mathrm{ns}}]$
LU Decomp. + SIMPLEC

$[S_{mag}]$
AMG

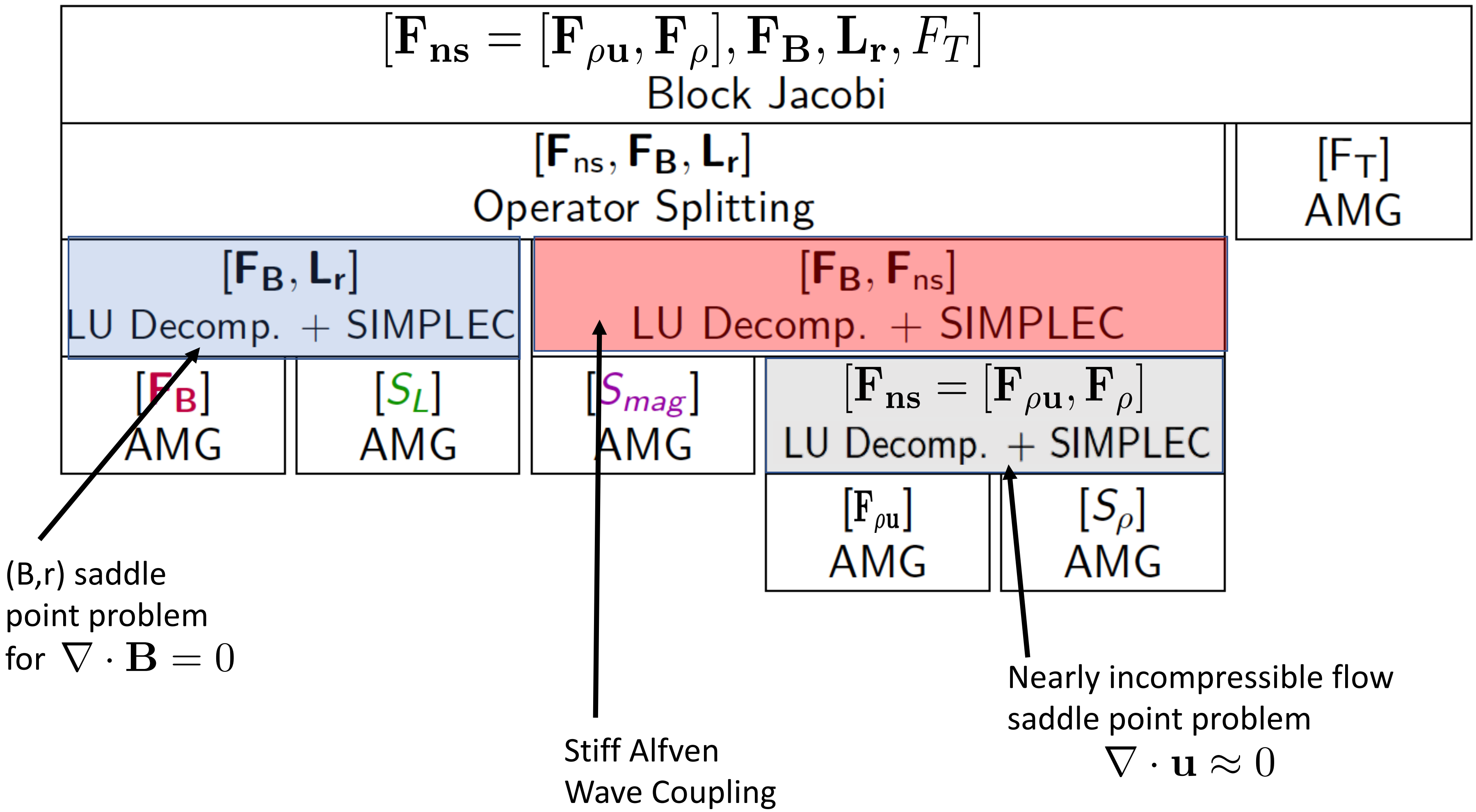
$[\mathbf{F}_{\mathrm{ns}}]$
LU Decomp. + SIMPLEC

$[\mathrm{F}_{\boldsymbol{u}}]$
AMG

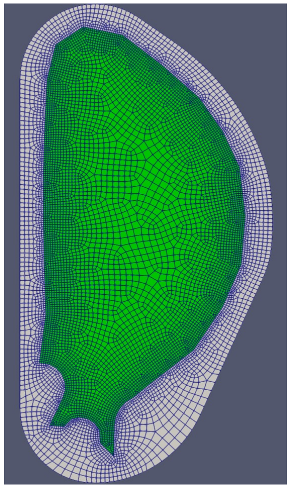
$[S_{\rho}]$
AMG

$$\mathbf{F}_{\mathrm{ns}} \approx \mathcal{M}_{NS} = \begin{bmatrix} \mathrm{F}_{\boldsymbol{\rho u}} & \tilde{\mathrm{B}}^{\mathrm{T}}_{\boldsymbol{\rho u},\rho} \\ & S_{\rho} \end{bmatrix}$$

$$S_{\rho} = \mathrm{F}_{\rho} - \mathrm{B}_{\rho,\boldsymbol{\rho u}}(\mathrm{absrowsum}(\mathrm{F}_{\boldsymbol{\rho u}}))^{-1}\,\tilde{\mathrm{B}}^{\mathrm{T}}_{\boldsymbol{\rho u},\rho}$$


$[\mathbf{F}_{\mathbf{ns}} = [\mathbf{F}_{\rho\mathbf{u}}, \mathbf{F}_{\rho}], \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}, F_T]$
Block Jacobi
$[\mathbf{F}_{\mathrm{ns}}, \mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}]$
Operator Splitting
$[\mathrm{F}_{\mathrm{T}}]$
AMG
$[\mathbf{F}_{\mathbf{B}}, \mathbf{L}_{\mathbf{r}}]$
LU Decomp. + SIMPLEC
$[\mathbf{F}_{\mathbf{B}}, \mathbf{F}_{\mathrm{ns}}]$
LU Decomp. + SIMPLEC
$[\mathbf{F}_{\mathbf{B}}]$
AMG
$[S_L]$
AMG
$[S_{mag}]$
AMG
$[\mathbf{F}_{\mathbf{ns}} = [\mathbf{F}_{\rho\mathbf{u}}, \mathbf{F}_{\rho}]$
LU Decomp. + SIMPLEC
$[\mathbf{F}_{\rho\mathbf{u}}]$
AMG
$[S_{\rho}]$
AMG
(B,r) saddle
point problem
for $\nabla \cdot \mathbf{B} = 0$
Stiff Alfven
Wave Coupling
Nearly incompressible flow
saddle point problem
$\nabla \cdot \mathbf{u} \approx 0$

Drekar Strong Scaling Results 3D ITER VDE

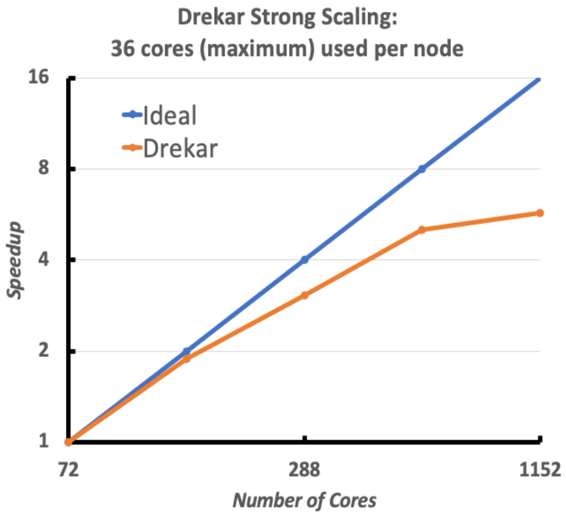


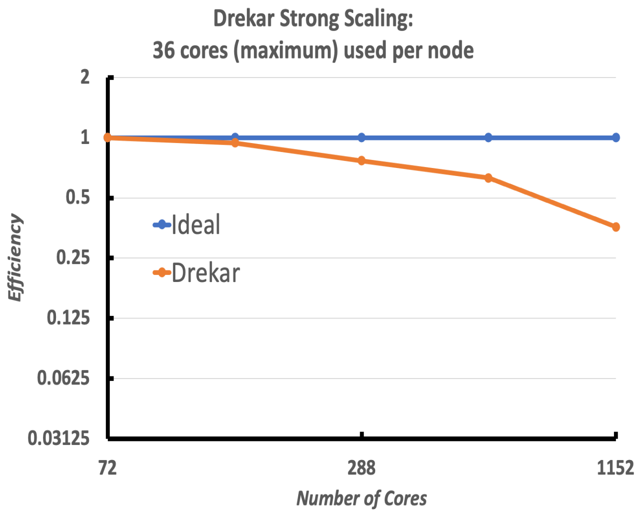
Strong scaling unperturbed initial equilibrium on coarse mesh with S = 1e4.

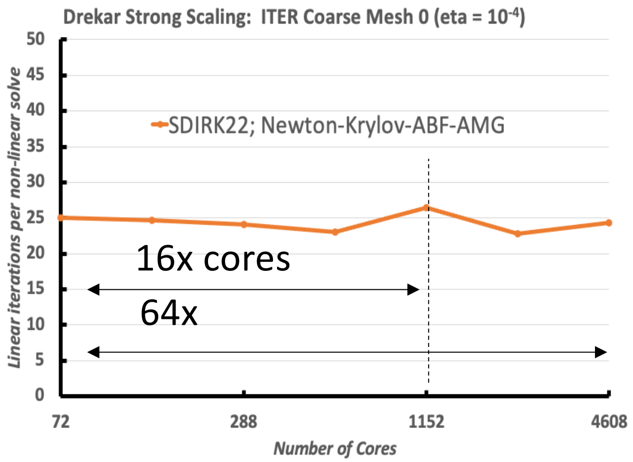
To 250 global Alfven times. 72 cores → 4608 cores (64x increase).

Coarse Mesh 0: 347K elements, 7.2K poloidal x 48 toroidal

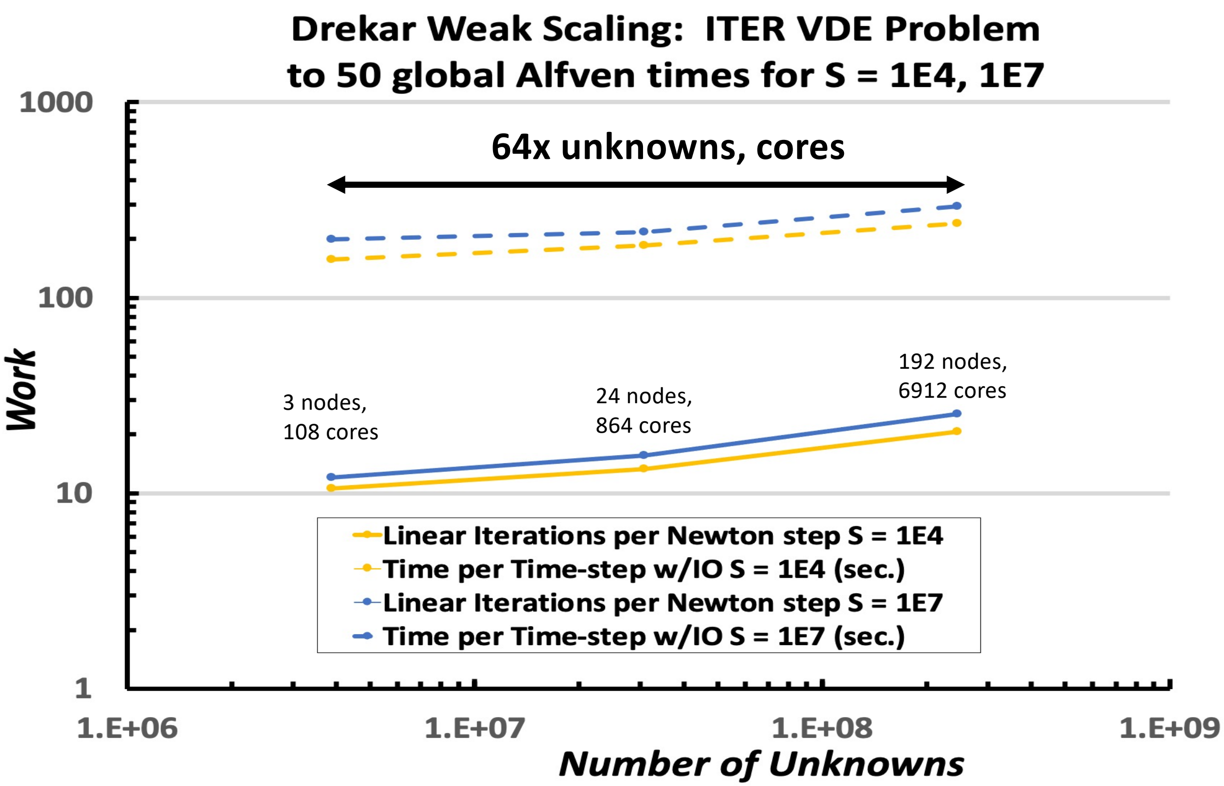
Constant time-step size dt = 2
Max CFL_A ~ 750, CFL_u ~3.2



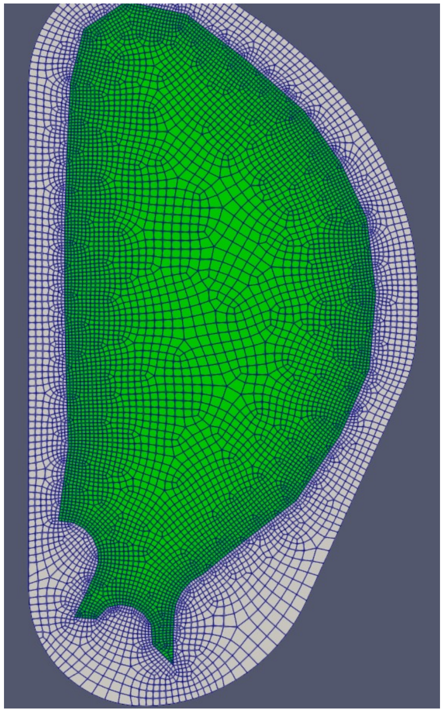




Drekar Weak scaling Study 3D ITER VDE




```
mesh 0. : 347K  elements,  7.2K poloidal x  48 toroidal, dt ~ 1.2;  108  cores on ghost
mesh 1. : 2.77M elements, 28.8K poloidal x  96 toroidal, dt ~ 0.55; 864  cores on ghost
mesh 2. : 22.2M elements,115.2K poloidal x 192 toroidal, dt ~ 0.25; 6912 cores on ghost
```



Unstructured mesh sequence

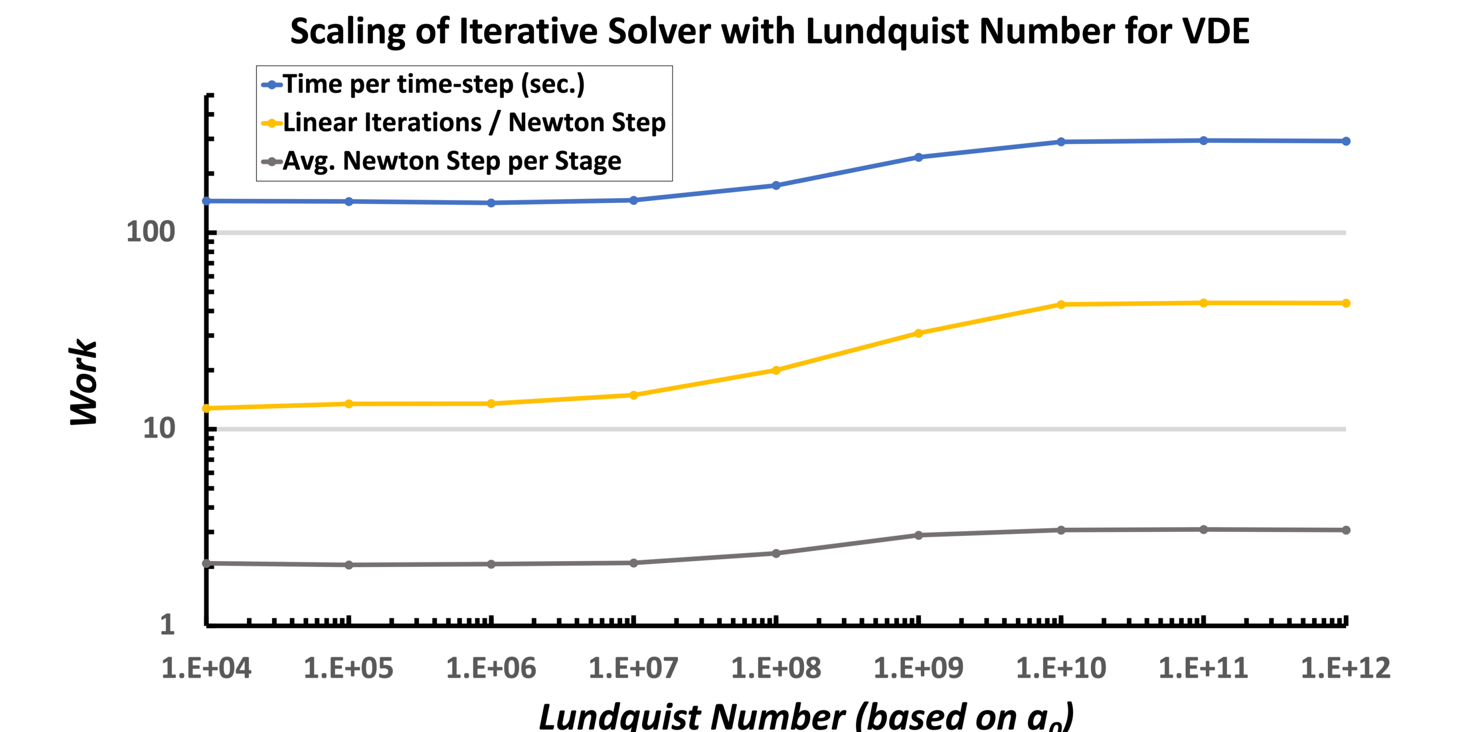
To 50 global Alfven times

Max CFL_A = 400, CFL_u = 2

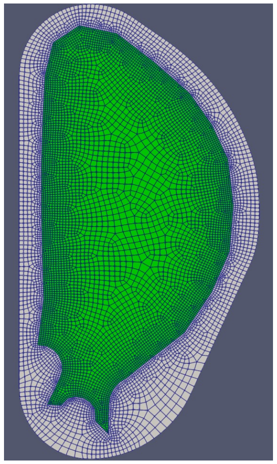
Pr_m = 0.1, Pr_T = 1

Lundquist Number scaling, coarse mesh 0.5 to 25 global Alfven times.

(Pr_m = 10, Pr_T = 1 i.e. both momentum and thermal diffusivities the same, scale 10x resistivity)



mesh 0.5: 1.28M elements, 1120 ARM cores



```
Mesh 0.5:
1.28M elements,10K
poloidal x 128
toroidal, 1120 ARM
cores
```

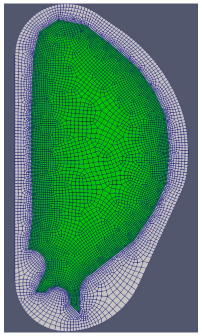
To 25 global Alfven times

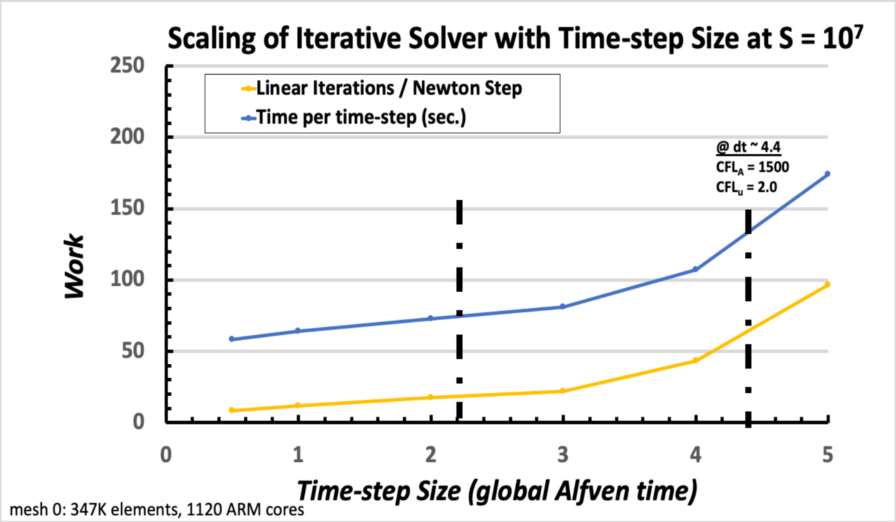
Max CFL_A = 400, CFL_u = 2

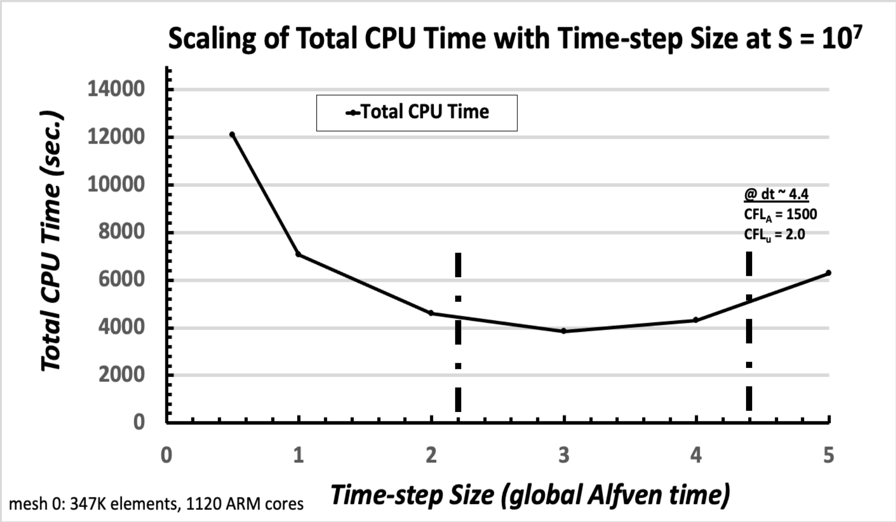
dt ~ 1.0

Time-step Size scaling for S = 10^7

Coarse mesh0







To 100 global Alfven times

Conclusions

- Developed scalable fully implicit low Mach compressible visco-resistive MHD solver.
  - Stabilized FE formulation → definite linear system and solvability (demonstated numerically)
  - Approximate block factorization → scalable treatment of multiphysics equation coupling.
  - Scalable AMG solves → efficient scalable sub-block preconditioning.
- Demonstrated scalability (strong and weak), Lundquist number robustness, promising initial efficiency for longer-time scale simulations.
- Proof-of-principle numerical experiments.
  - Cold VDE.
  - (1,1) internal kink mode.
- Future work
  - Further V&V benchmarks.
  - Extension to two temperature models (T_i, T_e).

Tokamak Disruption Simulation (TDS) SciDAC Center

Ten TDS partner institutions

Tokamak Disruption Simulation (TDS) SciDAC center

DOE Office of Science
ASCR/OFES SciDAC-4 Partnership (FY18 – FY23)

Xianzhu Tang (Lead – DOE PI, LANL PI)
J Shadid (DOE ASCR – PI, SNL PI)